1 STRAIGHT LINES AND LINEAR FUNCTIONS

PhotoDisc

Because the over-65 population will be growing more rapidly in the next few decades, U.S. health-care expenditures are expected to be boosted significantly. What will be the rate of increase of these expenditures over the next few years? How much will health-care expenditures be in 2014? In Example 1, page 29, we use a mathematical model based on figures from the Centers for Medicare and Medicaid to answer these questions.

THIS CHAPTER INTRODUCES the Cartesian coordinate system, a system that allows us to represent points in the plane in terms of ordered pairs of real numbers. This in turn enables us to compute the distance between two points algebraically. We also study straight lines. *Linear functions,* whose graphs are straight lines, can be used to describe many relationships between two quantities. These relationships can be found in fields of study as diverse as business, economics, the social sciences, physics, and medicine. In addition, we see how some practical problems can be solved by finding the point(s) of intersection of two straight lines. Finally, we learn how to find an algebraic representation of the straight line that "best" fits a set of data points that are scattered about a straight line.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 2

1.1 The Cartesian Coordinate System

The Cartesian Coordinate System

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division. We assume that you are familiar with the rules governing these algebraic operations (see Appendix B).

Real numbers may be represented geometrically by points on a line. This line is called the **real number,** or **coordinate, line.** We can construct the real number line as follows: Arbitrarily select a point on a straight line to represent the number 0. This point is called the **origin.** If the line is horizontal, then choose a point at a convenient distance to the right of the origin to represent the number 1. This determines the scale for the number line. Each positive real number x lies x units to the right of 0, and each negative real number x lies $-x$ units to the left of 0.

In this manner, a one-to-one correspondence is set up between the set of real numbers and the set of points on the number line, with all the positive numbers lying to the right of the origin and all the negative numbers lying to the left of the origin (Figure 1).

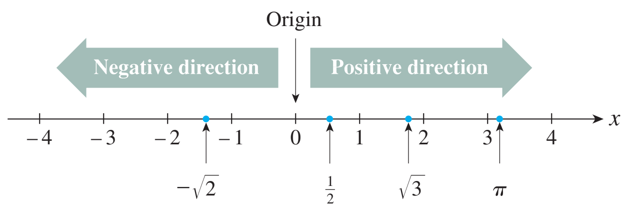


FIGURE 1
The real number line

In a similar manner, we can represent points in a plane (a two-dimensional space) by using the **Cartesian coordinate system,** which we construct as follows: Take two perpendicular lines, one of which is normally chosen to be horizontal. These lines intersect at a point O, called the **origin** (Figure 2). The horizontal line is called the ***x*-axis,** and the vertical line is called the ***y*-axis.** A number scale is set up along the x-axis, with the positive numbers lying to the right of the origin and the negative numbers lying to the left of it. Similarly, a number scale is set up along the y-axis, with the positive numbers lying above the origin and the negative numbers lying below it.

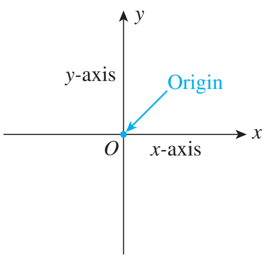


FIGURE 2
The Cartesian coordinate system

Note The number scales on the two axes need not be the same. Indeed, in many applications, different quantities are represented by x and y. For example, x may represent the number of cell phones sold, and y may represent the total revenue resulting from the sales. In such cases, it is often desirable to choose different number scales to represent the different quantities. Note, however, that the zeros of both number scales coincide at the origin of the two-dimensional coordinate system. ■

We can represent a point in the plane that is uniquely in this coordinate system by an **ordered pair** of numbers—that is, a pair (x, y) in which x is the first number and y is the second. To see this, let P be any point in the plane (Figure 3). Draw perpendicular lines from P to the x-axis and y-axis, respectively. Then the number x is precisely the number that corresponds to the point on the x-axis at which the perpendicular line through P hits the x-axis. Similarly, y is the number that corresponds to the point on the y-axis at which the perpendicular line through P crosses the y-axis.

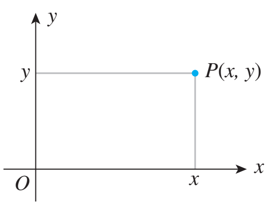


FIGURE 3
An ordered pair in the coordinate plane

Conversely, given an ordered pair (x, y) with x as the first number and y as the second, a point P in the plane is uniquely determined as follows: Locate the point on the x-axis represented by the number x, and draw a line through that point perpendicular to the x-axis. Next, locate the point on the y-axis represented by the number y, and draw a line through that point perpendicular to the y-axis. The point of intersection of these two lines is the point P (Figure 3).

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 3

In the ordered pair (x, y), x is called the **abscissa,** or **x-coordinate;** y is called the **ordinate,** or **y-coordinate;** and x and y together are referred to as the **coordinates** of the point P. The point P with x-coordinate equal to a and y-coordinate equal to b is often written $P(a, b)$.

The points $A(2, 3)$, $B(-2, 3)$, $C(-2, -3)$, $D(2, -3)$, $E(3, 2)$, $F(4, 0)$, and $G(0, -5)$ are plotted in Figure 4.

Note In general, $(x, y) \neq (y, x)$. This is illustrated by the points A and E in Figure 4.

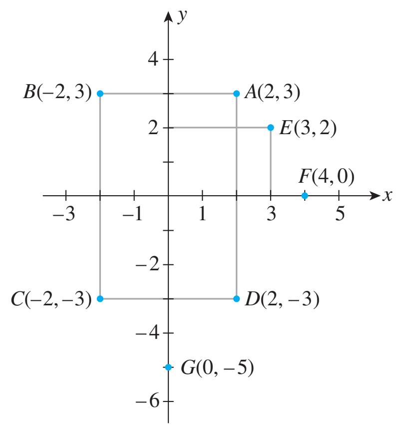


FIGURE 4
Several points in the coordinate plane

The axes divide the plane into four quadrants. Quadrant I consists of the points P with coordinates x and y, denoted by $P(x, y)$, satisfying $x > 0$ and $y > 0$; Quadrant II consists of the points $P(x, y)$ where $x < 0$ and $y > 0$; Quadrant III consists of the points $P(x, y)$ where $x < 0$ and $y < 0$; and Quadrant IV consists of the points $P(x, y)$ where $x > 0$ and $y < 0$ (Figure 5).

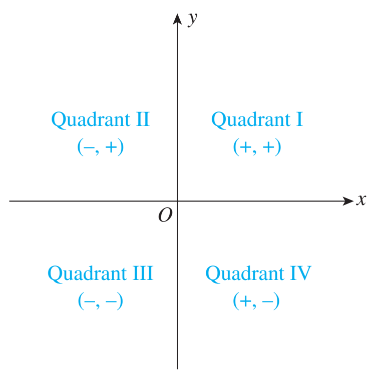


FIGURE 5
The four quadrants in the coordinate plane

The Distance Formula

One immediate benefit that arises from using the Cartesian coordinate system is that the distance between any two points in the plane may be expressed solely in terms of the coordinates of the points. Suppose, for example, (x_1, y_1) and (x_2, y_2) are any two points in the plane (Figure 6). Then we have the following:

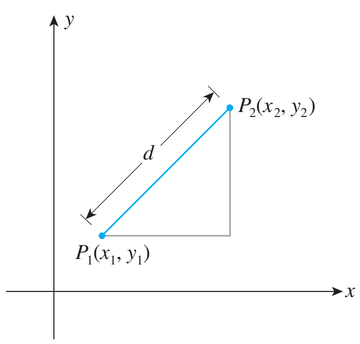


FIGURE 6
The distance between two points in the coordinate plane

Distance Formula

The distance d between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \textbf{(1)}$$

For a proof of this result, see Exercise 46, page 8.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 4

> ***Explore & Discuss***
>
> Refer to Example 1. Suppose we label the point (2, 6) as P_1 and the point $(-4, 3)$ as P_2. (1) Show that the distance d between the two points is the same as that obtained earlier. (2) Prove that, in general, the distance d in Formula (1) is independent of the way we label the two points.

In what follows, we give several applications of the distance formula.

EXAMPLE 1 Find the distance between the points $(-4, 3)$ and $(2, 6)$.

Solution Let $P_1(-4, 3)$ and $P_2(2, 6)$ be points in the plane. Then we have

$$x_1 = -4 \quad \text{and} \quad y_1 = 3$$
$$x_2 = 2 \qquad\qquad y_2 = 6$$

Using Formula (1), we have

$$\begin{aligned} d &= \sqrt{[2 - (-4)]^2 + (6 - 3)^2} \\ &= \sqrt{6^2 + 3^2} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

■

APPLIED EXAMPLE 2 The Cost of Laying Cable In Figure 7, S represents the position of a power relay station located on a straight coastal highway, and M shows the location of a marine biology experimental station on a nearby island. A cable is to be laid connecting the relay station at S with the experimental station at M via the point Q that lies on the x-axis between O and S. If the cost of running the cable on land is \$3 per running foot and the cost of running the cable underwater is \$5 per running foot, find the total cost for laying the cable.

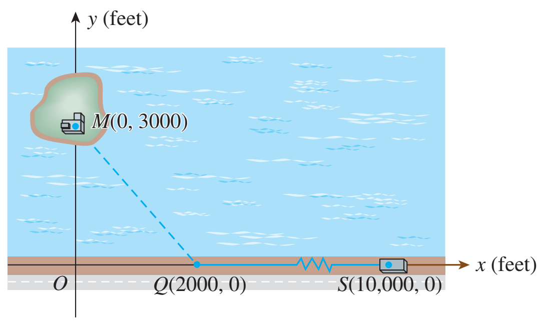


FIGURE 7
The cable will connect the relay station S to the experimental station M.

Solution The length of cable required on land is given by the distance from S to Q. This distance is $(10{,}000 - 2000)$, or 8000 feet. Next, we see that the length of cable required underwater is given by the distance from Q to M. This distance is

$$\begin{aligned} \sqrt{(0 - 2000)^2 + (3000 - 0)^2} &= \sqrt{2000^2 + 3000^2} \\ &= \sqrt{13{,}000{,}000} \\ &\approx 3606 \end{aligned}$$

or approximately 3606 feet. Therefore, the total cost for laying the cable is approximately

$$3(8000) + 5(3606) \approx 42{,}030$$

dollars. ■

VIDEO

EXAMPLE 3 Let $P(x, y)$ denote a point lying on a circle with radius r and center $C(h, k)$ (Figure 8). Find a relationship between x and y.

Solution By the definition of a circle, the distance between $C(h, k)$ and $P(x, y)$ is r. Using Formula (1), we have

$$\sqrt{(x - h)^2 + (y - k)^2} = r$$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 5

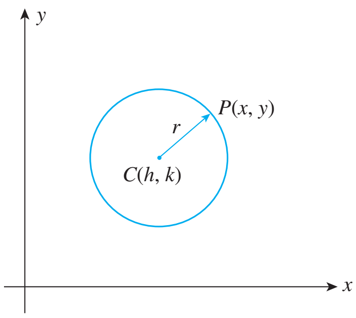


FIGURE 8
A circle with radius r and center $C(h, k)$

which, upon squaring both sides, gives the equation

$$(x - h)^2 + (y - k)^2 = r^2$$

which must be satisfied by the variables x and y.

A summary of the result obtained in Example 3 follows.

Equation of a Circle

An equation of the circle with center $C(h, k)$ and radius r is given by

$$(x - h)^2 + (y - k)^2 = r^2 \qquad \textbf{(2)}$$

EXAMPLE 4 Find an equation of the circle with (a) radius 2 and center $(-1, 3)$ and (b) radius 3 and center located at the origin.

Solution

a. We use Formula (2) with $r = 2$, $h = -1$, and $k = 3$, obtaining

$$[x - (-1)]^2 + (y - 3)^2 = 2^2$$
$$(x + 1)^2 + (y - 3)^2 = 4$$

(Figure 9a).

b. Using Formula (2) with $r = 3$ and $h = k = 0$, we obtain

$$x^2 + y^2 = 3^2$$
$$x^2 + y^2 = 9$$

(Figure 9b).

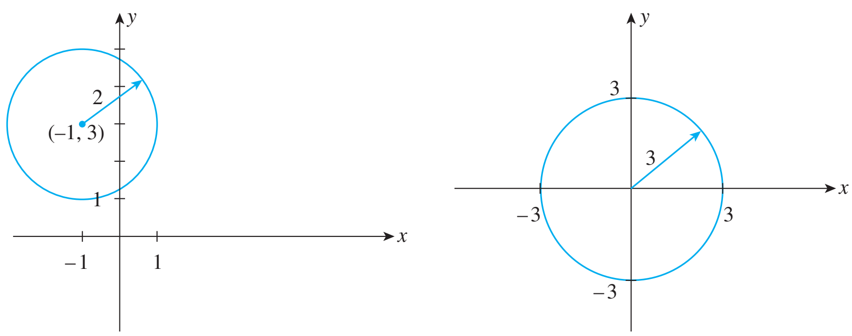


FIGURE 9

(a) The circle with radius 2 and center $(-1, 3)$

(b) The circle with radius 3 and center (0, 0)

Explore & Discuss

1. Use the distance formula to help you describe the set of points in the xy-plane satisfying each of the following inequalities, where $r > 0$.

a. $(x - h)^2 + (y - k)^2 \leq r^2$ **c.** $(x - h)^2 + (y - k)^2 \geq r^2$

b. $(x - h)^2 + (y - k)^2 < r^2$ **d.** $(x - h)^2 + (y - k)^2 > r^2$

2. Consider the equation $x^2 + y^2 = 4$.

a. Show that $y = \pm\sqrt{4 - x^2}$.

b. Describe the set of points (x, y) in the xy-plane satisfying the equation

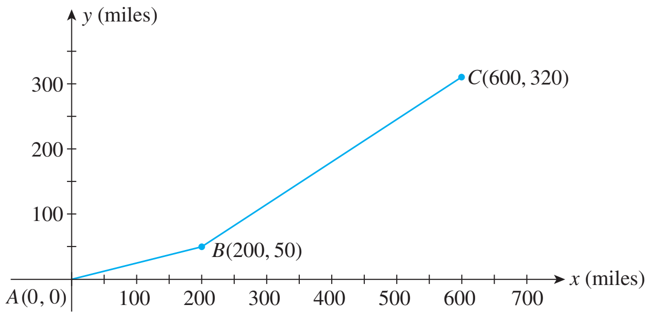
(i) $y = \sqrt{4 - x^2}$ (ii) $y = -\sqrt{4 - x^2}$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 6

1.1 Self-Check Exercises

1. **a.** Plot the points $A(4, -2)$, $B(2, 3)$, and $C(-3, 1)$.
   b. Find the distance between the points A and B, between B and C, and between A and C.
   c. Use the Pythagorean Theorem to show that the triangle with vertices A, B, and C is a right triangle.

2. The accompanying figure shows the location of Cities A, B, and C. Suppose a pilot wishes to fly from City A to City C but must make a mandatory stopover in City B. If the single-engine light plane has a range of 650 mi, can the pilot make the trip without refueling in City B?



Solutions to Self-Check Exercises 1.1 can be found on page 9.

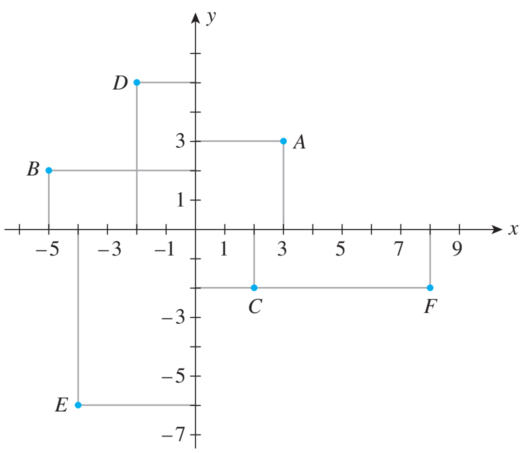
1.1 Concept Questions

1. What can you say about the signs of a and b if the point $P(a, b)$ lies in (a) the second quadrant? (b) The third quadrant? (c) The fourth quadrant?

2. **a.** What is the distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$?
   b. When you use the distance formula, does it matter which point is labeled P_1 and which point is labeled P_2? Explain.

1.1 Exercises

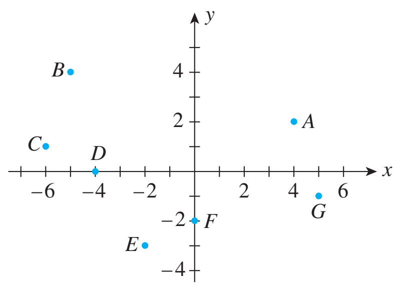
In Exercises 1–6, refer to the accompanying figure and determine the coordinates of the point and the quadrant in which it is located.



1. A **2.** B **3.** C

4. D **5.** E **6.** F

In Exercises 7–12, refer to the accompanying figure.



7. Which point has coordinates (4, 2)?

8. What are the coordinates of point B?

9. Which points have negative y-coordinates?

10. Which point has a negative x-coordinate and a negative y-coordinate?

11. Which point has an x-coordinate that is equal to zero?

12. Which point has a y-coordinate that is equal to zero?

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 7

In Exercises 13–20, sketch a set of coordinate axes and then plot the point.

13. $(-2, 5)$
14. $(1, 3)$
15. $(3, -1)$
16. $(3, -4)$
17. $(8, -\frac{7}{2})$
18. $(-\frac{5}{2}, \frac{3}{2})$
19. $(4.5, -4.5)$
20. $(1.2, -3.4)$

In Exercises 21–24, find the distance between the points.

21. $(1, 3)$ and $(4, 7)$
22. $(1, 0)$ and $(4, 4)$
23. $(-1, 3)$ and $(4, 9)$
24. $(-2, 1)$ and $(10, 6)$

25. Find the coordinates of the points that are 10 units away from the origin and have a y-coordinate equal to -6.

26. Find the coordinates of the points that are 5 units away from the origin and have an x-coordinate equal to 3.

27. Show that the points $(3, 4)$, $(-3, 7)$, $(-6, 1)$, and $(0, -2)$ form the vertices of a square.

28. Show that the triangle with vertices $(-5, 2)$, $(-2, 5)$, and $(5, -2)$ is a right triangle.

In Exercises 29–34, find an equation of the circle that satisfies the given conditions.

29. Radius 5 and center $(2, -3)$

30. Radius 3 and center $(-2, -4)$

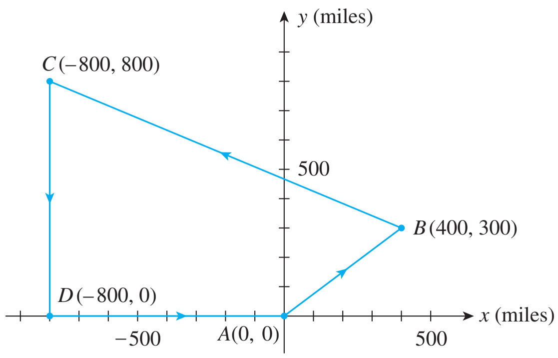
31. Radius 5 and center at the origin

32. Center at the origin and passes through $(2, 3)$

33. Center $(2, -3)$ and passes through $(5, 2)$

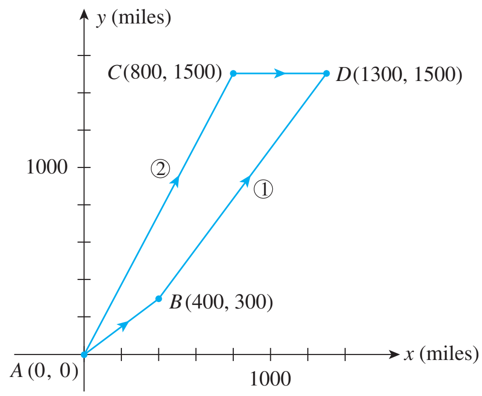
34. Center $(-a, a)$ and radius $2a$

35. DISTANCE TRAVELED A grand tour of four cities begins at City A and makes successive stops at Cities B, C, and D before returning to City A. If the cities are located as shown in the accompanying figure, find the total distance covered on the tour.



36. DELIVERY CHARGES A furniture store offers free setup and delivery services to all points within a 25-mi radius of its warehouse distribution center. If you live 20 mi east and 14 mi south of the warehouse, will you incur a delivery charge? Justify your answer.

37. OPTIMIZING TRAVEL TIME Towns A, B, C, and D are located as shown in the accompanying figure. Two highways link Town A to Town D. Route 1 runs from Town A to Town D via Town B, and Route 2 runs from Town A to Town D via Town C. If a salesman wishes to drive from Town A to Town D and traffic conditions are such that he could expect to average the same speed on either route, which highway should he take to arrive in the shortest time?



38. MINIMIZING SHIPPING COSTS Refer to the figure for Exercise 37. Suppose a fleet of 100 automobiles are to be shipped from an assembly plant in Town A to Town D. They may be shipped either by freight train along Route 1 at a cost of 66¢/mile/automobile or by truck along Route 2 at a cost of 62¢/mile/automobile. Which means of transportation minimizes the shipping cost? What is the net savings?

39. CONSUMER DECISIONS Will Barclay wishes to determine which HDTV antenna he should purchase for his home. The TV store has supplied him with the following information:

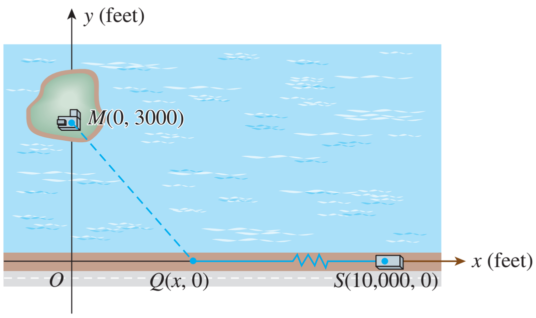
Range in miles			
VHF	**UHF**	**Model**	**Price**
30	20	A	$50
45	35	B	60
60	40	C	70
75	55	D	80

Will wishes to receive Channel 17 (VHF), which is located 25 mi east and 35 mi north of his home, and Channel 38 (UHF), which is located 20 mi south and 32 mi west of his home. Which model will allow him to receive both channels at the least cost? (Assume that the terrain between Will's home and both broadcasting stations is flat.)

40. COST OF LAYING CABLE In the accompanying diagram, S represents the position of a power relay station located on a straight coastal highway, and M shows the location of a marine biology experimental station on a nearby island. A

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 8

cable is to be laid connecting the relay station at S with the experimental station at M via the point Q that lies on the x-axis between O and S. If the cost of running the cable on land is \$3/running foot and the cost of running cable underwater is \$5/running foot, find an expression in terms of x that gives the total cost of laying the cable. Use this expression to find the total cost when $x = 1500$ and when $x = 2500$.



41. Distance Between Ships Two ships leave port at the same time. Ship A sails north at a speed of 20 mph while Ship B sails east at a speed of 30 mph.

a. Find an expression in terms of the time t (in hours) giving the distance between the two ships.

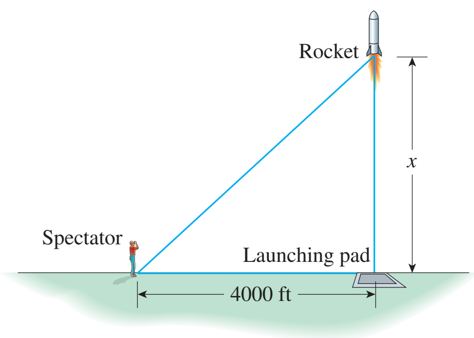
b. Using the expression obtained in part (a), find the distance between the two ships 2 hr after leaving port.

42. Distance Between Ships Sailing north at a speed of 25 mph, Ship A leaves a port. A half hour later, Ship B leaves the same port, sailing east at a speed of 20 mph. Let t (in hours) denote the time Ship B has been at sea.

a. Find an expression in terms of t that gives the distance between the two ships.

b. Use the expression obtained in part (a) to find the distance between the two ships 2 hr after Ship A has left the port.

43. Watching a Rocket Launch At a distance of 4000 ft from the launch site, a spectator is observing a rocket being launched. Suppose the rocket lifts off vertically and reaches an altitude of x feet, as shown below:



a. Find an expression giving the distance between the spectator and the rocket.

b. What is the distance between the spectator and the rocket when the rocket reaches an altitude of 20,000 ft?

In Exercises 44 and 45, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

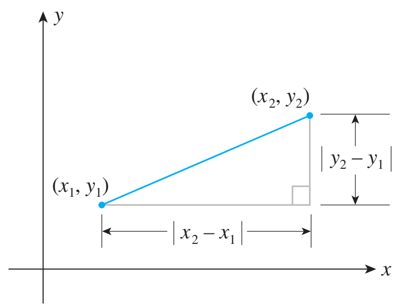
44. If the distance between the points $P_1(a, b)$ and $P_2(c, d)$ is D, then the distance between the points $P_1(a, b)$ and $P_3(kc, kd)$ $(k \neq 0)$ is given by $|k|D$.

45. The circle with equation $kx^2 + ky^2 = a^2$ lies inside the circle with equation $x^2 + y^2 = a^2$, provided that $k > 1$ and $a > 0$.

46. Let (x_1, y_1) and (x_2, y_2) be two points lying in the xy-plane. Show that the distance between the two points is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Hint: Refer to the accompanying figure, and use the Pythagorean Theorem.



47. a. Show that the midpoint of the line segment joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

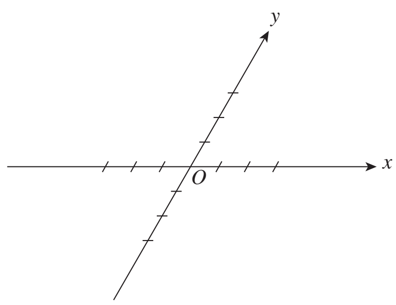
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

b. Use the result of part (a) to find the midpoint of the line segment joining the points $(-3, 2)$ and $(4, -5)$.

48. In the Cartesian coordinate system, the two axes are perpendicular to each other. Consider a coordinate system in which the x-axis and y-axis are noncollinear (that is, the axes do not lie along a straight line) and are not perpendicular to each other (see the accompanying figure).

a. Describe how a point is represented in this coordinate system by an ordered pair (x, y) of real numbers. Conversely, show how an ordered pair (x, y) of real numbers uniquely determines a point in the plane.

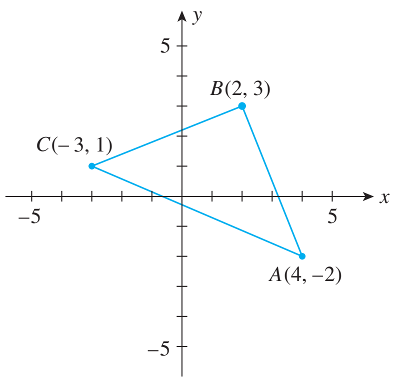
b. Suppose you want to find a formula for the distance between two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, in the plane. What advantage does the Cartesian coordinate system have over the coordinate system under consideration?



48149_01_ch1_p001-066 11/2/10 10:35 AM Page 9

1.1 Solutions to Self-Check Exercises

1. a. The points are plotted in the following figure.



b. The distance between A and B is

$$\begin{aligned} d(A, B) &= \sqrt{(2-4)^2 + [3-(-2)]^2} \\ &= \sqrt{(-2)^2 + 5^2} = \sqrt{4+25} = \sqrt{29} \end{aligned}$$

The distance between B and C is

$$\begin{aligned} d(B, C) &= \sqrt{(-3-2)^2 + (1-3)^2} \\ &= \sqrt{(-5)^2 + (-2)^2} = \sqrt{25+4} = \sqrt{29} \end{aligned}$$

The distance between A and C is

$$\begin{aligned} d(A, C) &= \sqrt{(-3-4)^2 + [1-(-2)]^2} \\ &= \sqrt{(-7)^2 + 3^2} = \sqrt{49+9} = \sqrt{58} \end{aligned}$$

c. We will show that

$$[d(A, C)]^2 = [d(A, B)]^2 + [d(B, C)]^2$$

From part (b), we see that $[d(A, B)]^2 = 29$, $[d(B, C)]^2 = 29$, and $[d(A, C)]^2 = 58$, and the desired result follows.

2. The distance between City A and City B is

$$d(A, B) = \sqrt{200^2 + 50^2} \approx 206$$

or 206 mi. The distance between City B and City C is

$$\begin{aligned} d(B, C) &= \sqrt{(600-200)^2 + (320-50)^2} \\ &= \sqrt{400^2 + 270^2} \approx 483 \end{aligned}$$

or 483 mi. Therefore, the total distance the pilot would have to cover is 689 mi, so she must refuel in City B.

1.2 Straight Lines

In computing income tax, business firms are allowed by law to depreciate certain assets such as buildings, machines, furniture, and automobiles over a period of time. *Linear depreciation*, or the *straight-line method*, is often used for this purpose. The graph of the straight line shown in Figure 10 describes the book value V of a network server that has an initial value of \$10,000 and that is being depreciated linearly over 5 years with a scrap value of \$3000. Note that only the solid portion of the straight line is of interest here.

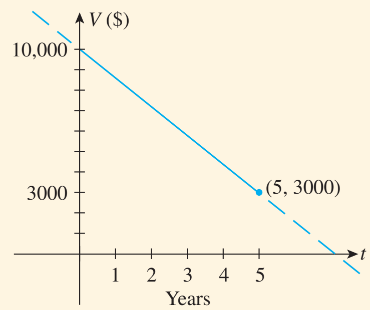


FIGURE **10**
Linear depreciation of an asset

The book value of the server at the end of year t, where t lies between 0 and 5, can be read directly from the graph. But there is one shortcoming in this approach: The result depends on how accurately you draw and read the graph. A better and more accurate method is based on finding an *algebraic* representation of the depreciation line. (We continue our discussion of the linear depreciation problem in Section 1.3.)

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 10

To see how a straight line in the *xy*-plane may be described algebraically, we need first to recall certain properties of straight lines.

Slope of a Line

Let L denote the unique straight line that passes through the two distinct points (x_1, y_1) and (x_2, y_2). If $x_1 \neq x_2$, then we define the slope of L as follows.

Slope of a Nonvertical Line

If (x_1, y_1) and (x_2, y_2) are any two distinct points on a nonvertical line L, then the slope m of L is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \textbf{(3)}$$

(Figure 11).

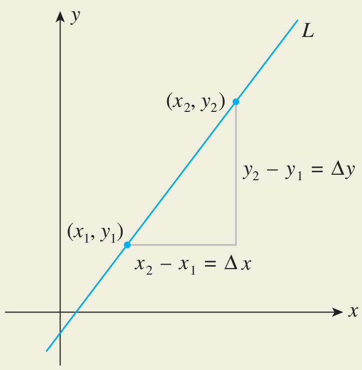


FIGURE **11**

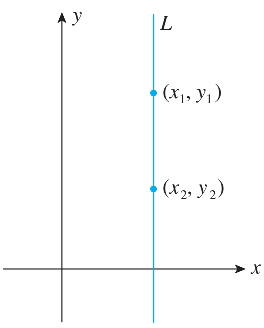


FIGURE **12**
The slope is undefined if $x_1 = x_2$.

If $x_1 = x_2$, then L is a vertical line (Figure 12). Its slope is undefined since the denominator in Equation (3) will be zero and division by zero is proscribed.

Observe that the slope of a straight line is a constant whenever it is defined. The number $\Delta y = y_2 - y_1$ (Δy is read "delta y") is a measure of the vertical change in y, and $\Delta x = x_2 - x_1$ is a measure of the horizontal change in x as shown in Figure 11. From this figure we can see that the slope m of a straight line L is a measure of the *rate of change of y with respect to x*. Furthermore, the slope of a nonvertical straight line is constant, and this tells us that this rate of change is constant.

Figure 13a shows a straight line L_1 with slope 2. Observe that L_1 has the property that a 1-unit increase in x results in a 2-unit increase in y. To see this, let $\Delta x = 1$ in

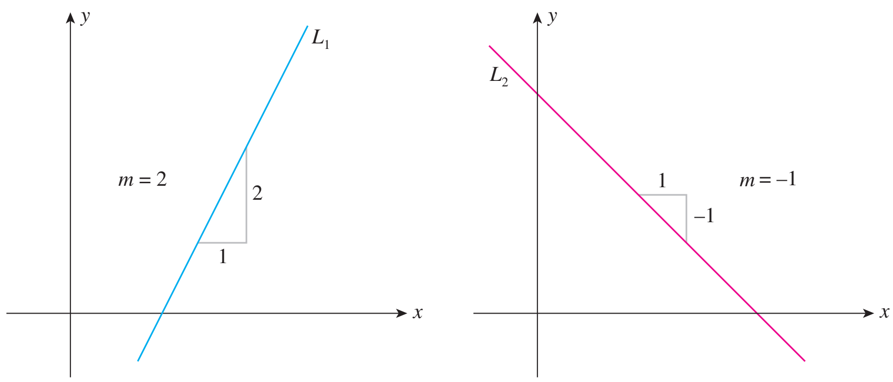


FIGURE **13**

(a) The line rises ($m > 0$).

(b) The line falls ($m < 0$).

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 11

Equation (3) so that $m = \Delta y$. Since $m = 2$, we conclude that $\Delta y = 2$. Similarly, Figure 13b shows a line L_2 with slope -1. Observe that a straight line with positive slope slants upward from left to right (y increases as x increases), whereas a line with negative slope slants downward from left to right (y decreases as x increases). Finally, Figure 14 shows a family of straight lines passing through the origin with indicated slopes.

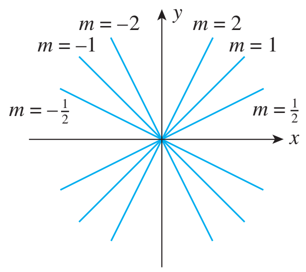


FIGURE **14**
A family of straight lines

Explore & Discuss

Show that the slope of a nonvertical line is independent of the two distinct points used to compute it.

Hint: Suppose we pick two other distinct points, $P_3(x_3, y_3)$ and $P_4(x_4, y_4)$ lying on L. Draw a picture, and use similar triangles to demonstrate that using P_3 and P_4 gives the same value as that obtained by using P_1 and P_2.

EXAMPLE 1 Sketch the straight line that passes through the point $(-2, 5)$ and has slope $-\frac{4}{3}$.

Solution First, plot the point $(-2, 5)$ (Figure 15). Next, recall that a slope of $-\frac{4}{3}$ indicates that an increase of 1 unit in the x-direction produces a *decrease* of $\frac{4}{3}$ units in the y-direction, or equivalently, a 3-unit increase in the x-direction produces a $3(\frac{4}{3})$, or 4-unit, decrease in the y-direction. Using this information, we plot the point $(1, 1)$ and draw the line through the two points.

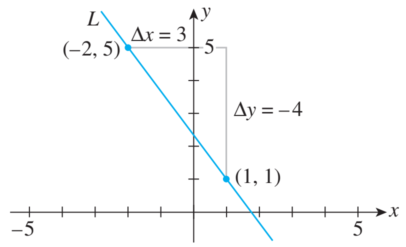


FIGURE **15**
L has slope $-\frac{4}{3}$ and passes through $(-2, 5)$.

EXAMPLE 2 Find the slope m of the line that passes through the points $(-1, 1)$ and $(5, 3)$.

Solution Choose (x_1, y_1) to be the point $(-1, 1)$ and (x_2, y_2) to be the point $(5, 3)$. Then, with $x_1 = -1$, $y_1 = 1$, $x_2 = 5$, and $y_2 = 3$, we find, using Equation (3),

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - (-1)} = \frac{2}{6} = \frac{1}{3}$$

(Figure 16). You may verify that the result obtained would be the same had we chosen the point $(-1, 1)$ to be (x_2, y_2) and the point $(5, 3)$ to be (x_1, y_1).

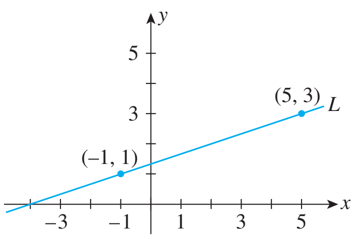


FIGURE **16**
L passes through $(5, 3)$ and $(-1, 1)$.

EXAMPLE 3 Find the slope of the line that passes through the points $(-2, 5)$ and $(3, 5)$.

Solution The slope of the required line is given by

$$m = \frac{5 - 5}{3 - (-2)} = \frac{0}{5} = 0$$

(Figure 17).

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 12

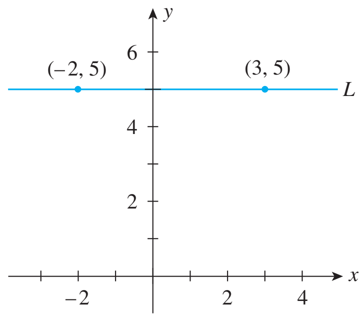


FIGURE 17
The slope of the horizontal line L is zero.

Note The slope of a horizontal line is zero.

We can use the slope of a straight line to determine whether a line is parallel to another line.

Parallel Lines

Two distinct lines are **parallel** if and only if their slopes are equal or their slopes are undefined.

EXAMPLE 4 Let L_1 be a line that passes through the points $(-2, 9)$ and $(1, 3)$, and let L_2 be the line that passes through the points $(-4, 10)$ and $(3, -4)$. Determine whether L_1 and L_2 are parallel.

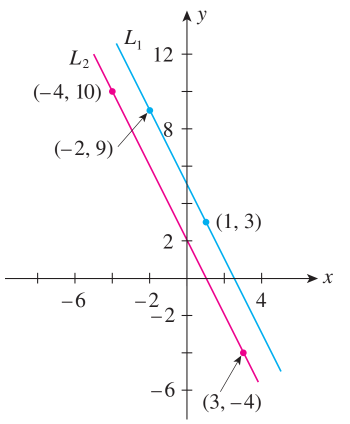


FIGURE 18
L_1 and L_2 have the same slope and hence are parallel.

Solution The slope m_1 of L_1 is given by

$$m_1 = \frac{3 - 9}{1 - (-2)} = -2$$

The slope m_2 of L_2 is given by

$$m_2 = \frac{-4 - 10}{3 - (-4)} = -2$$

Since $m_1 = m_2$, the lines L_1 and L_2 are in fact parallel (Figure 18).

Equations of Lines

We now show that every straight line lying in the xy-plane may be represented by an equation involving the variables x and y. One immediate benefit of this is that problems involving straight lines may be solved algebraically.

Let L be a straight line parallel to the y-axis (perpendicular to the x-axis) (Figure 19). Then L crosses the x-axis at some point $(a, 0)$ with the x-coordinate given by $x = a$, where a is some real number. Any other point on L has the form (a, y), where y is an appropriate number. Therefore, the vertical line L is described by the sole condition

$$x = a$$

and this is accordingly an equation of L. For example, the equation $x = -2$ represents a vertical line 2 units to the left of the y-axis, and the equation $x = 3$ represents a vertical line 3 units to the right of the y-axis (Figure 20).

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 13

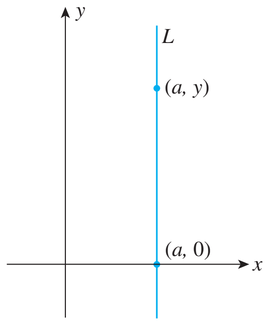


FIGURE 19
The vertical line $x = a$

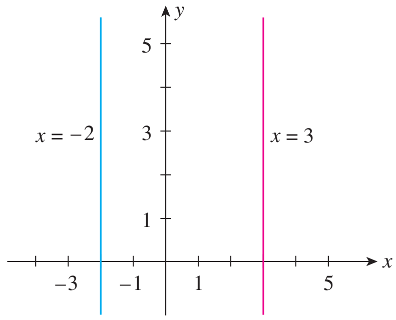


FIGURE 20
The vertical lines $x = -2$ and $x = 3$

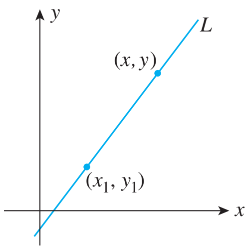


FIGURE 21
L passes through (x_1, y_1) and has slope m.

Next, suppose L is a nonvertical line, so it has a well-defined slope m. Suppose (x_1, y_1) is a fixed point lying on L and (x, y) is a variable point on L distinct from (x_1, y_1) (Figure 21). Using Equation (3) with the point $(x_2, y_2) = (x, y)$, we find that the slope of L is given by

$$m = \frac{y - y_1}{x - x_1}$$

Upon multiplying both sides of the equation by $x - x_1$, we obtain Equation (4).

Point-Slope Form of an Equation of a Line

An equation of the line that has slope m and passes through the point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1) \qquad \textbf{(4)}$$

Equation (4) is called the *point-slope form* of an equation of a line because it uses a given point (x_1, y_1) on a line and the slope m of the line.

EXAMPLE 5 Find an equation of the line that passes through the point (1, 3) and has slope 2.

Solution Using the point-slope form of the equation of a line with the point (1, 3) and $m = 2$, we obtain

$$y - 3 = 2(x - 1) \qquad y - y_1 = m(x - x_1)$$

which, when simplified, becomes

$$2x - y + 1 = 0$$

(Figure 22).

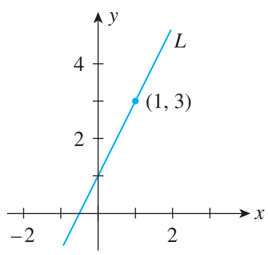


FIGURE 22
L passes through (1, 3) and has slope 2.

■

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 14

VIDEO **EXAMPLE 6** Find an equation of the line that passes through the points $(-3, 2)$ and $(4, -1)$.

Solution The slope of the line is given by

$$m = \frac{-1 - 2}{4 - (-3)} = -\frac{3}{7}$$

Using the point-slope form of the equation of a line with the point $(4, -1)$ and the slope $m = -\frac{3}{7}$, we have

$$\begin{aligned} y + 1 &= -\frac{3}{7}(x - 4) \qquad y - y_1 = m(x - x_1) \\ 7y + 7 &= -3x + 12 \\ 3x + 7y - 5 &= 0 \end{aligned}$$

(Figure 23).

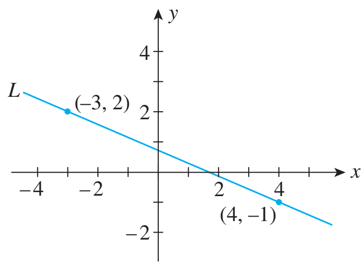


FIGURE **23**
L passes through $(-3, 2)$ and $(4, -1)$.

We can use the slope of a straight line to determine whether a line is perpendicular to another line.

Perpendicular Lines

If L_1 and L_2 are two distinct nonvertical lines that have slopes m_1 and m_2, respectively, then L_1 is **perpendicular** to L_2 (written $L_1 \perp L_2$) if and only if

$$m_1 = -\frac{1}{m_2}$$

If the line L_1 is vertical (so that its slope is undefined), then L_1 is perpendicular to another line, L_2, if and only if L_2 is horizontal (so that its slope is zero). For a proof of these results, see Exercise 92, page 22.

EXAMPLE 7 Find an equation of the line that passes through the point $(3, 1)$ and is perpendicular to the line of Example 5.

Solution Since the slope of the line in Example 5 is 2, it follows that the slope of the required line is given by $m = -\frac{1}{2}$, the negative reciprocal of 2. Using the point-slope form of the equation of a line, we obtain

$$\begin{aligned} y - 1 &= -\frac{1}{2}(x - 3) \qquad y - y_1 = m(x - x_1) \\ 2y - 2 &= -x + 3 \\ x + 2y - 5 &= 0 \end{aligned}$$

(Figure 24).

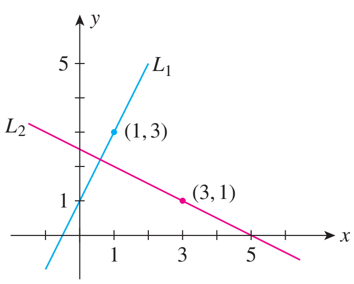


FIGURE **24**
L_2 is perpendicular to L_1 and passes through $(3, 1)$.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 15

Exploring with TECHNOLOGY

1. Use a graphing utility to plot the straight lines L_1 and L_2 with equations $2x + y - 5 = 0$ and $41x + 20y - 11 = 0$ on the same set of axes, using the standard viewing window.
   a. Can you tell whether the lines L_1 and L_2 are parallel to each other?
   b. Verify your observations by computing the slopes of L_1 and L_2 algebraically.
2. Use a graphing utility to plot the straight lines L_1 and L_2 with equations $x + 2y - 5 = 0$ and $5x - y + 5 = 0$ on the same set of axes, using the standard viewing window.
   a. Can you tell whether the lines L_1 and L_2 are perpendicular to each other?
   b. Verify your observation by computing the slopes of L_1 and L_2 algebraically.

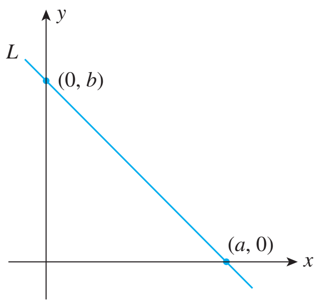


FIGURE 25
The line L has x-intercept a and y-intercept b.

A straight line L that is neither horizontal nor vertical cuts the x-axis and the y-axis at, say, points $(a, 0)$ and $(0, b)$, respectively (Figure 25). The numbers a and b are called the ***x*-intercept** and ***y*-intercept,** respectively, of L.

Now, let L be a line with slope m and y-intercept b. Using Equation (4), the point-slope form of the equation of a line, with the point given by $(0, b)$ and slope m, we have

$$y - b = m(x - 0)$$
$$y = mx + b$$

Slope-Intercept Form of an Equation of a Line

The equation of the line that has slope m and intersects the y-axis at the point $(0, b)$ is given by

$$y = mx + b \qquad \textbf{(5)}$$

EXAMPLE 8 Find an equation of the line that has slope 3 and y-intercept -4.

Solution Using Equation (5) with $m = 3$ and $b = -4$, we obtain the required equation:

$$y = 3x - 4$$

EXAMPLE 9 Determine the slope and y-intercept of the line whose equation is $3x - 4y = 8$.

Solution Rewrite the given equation in the slope-intercept form. Thus,

$$3x - 4y = 8$$
$$-4y = -3x + 8$$
$$y = \frac{3}{4}x - 2$$

Comparing this result with Equation (5), we find $m = \frac{3}{4}$ and $b = -2$, and we conclude that the slope and y-intercept of the given line are $\frac{3}{4}$ and -2, respectively.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 16

Exploring with TECHNOLOGY

1. Use a graphing utility to plot the straight lines with equations $y = -2x + 3$, $y = -x + 3$, $y = x + 3$, and $y = 2.5x + 3$ on the same set of axes, using the standard viewing window. What effect does changing the coefficient m of x in the equation $y = mx + b$ have on its graph?
2. Use a graphing utility to plot the straight lines with equations $y = 2x - 2$, $y = 2x - 1$, $y = 2x$, $y = 2x + 1$, and $y = 2x + 4$ on the same set of axes, using the standard viewing window. What effect does changing the constant b in the equation $y = mx + b$ have on its graph?
3. Describe in words the effect of changing both m and b in the equation $y = mx + b$.

VIDEO

APPLIED EXAMPLE 10 Sales of a Sporting Goods Store The sales manager of a local sporting goods store plotted sales versus time for the last 5 years and found the points to lie approximately along a straight line (Figure 26). By using the points corresponding to the first and fifth years, find an equation of the *trend line*. What sales figure can be predicted for the sixth year?

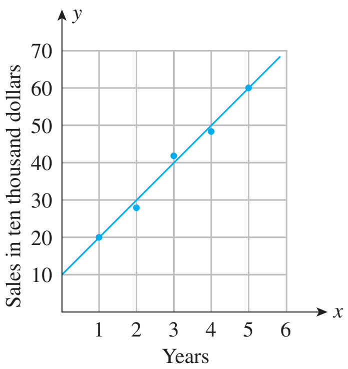


FIGURE 26
Sales of a sporting goods store

Solution Using Equation (3) with the points (1, 20) and (5, 60), we find that the slope of the required line is given by

$$m = \frac{60 - 20}{5 - 1} = 10$$

Next, using the point-slope form of the equation of a line with the point (1, 20) and $m = 10$, we obtain

$$y - 20 = 10(x - 1) \qquad y - y_1 = m(x - x_1)$$
$$y = 10x + 10$$

as the required equation.

The sales figure for the sixth year is obtained by letting $x = 6$ in the last equation, giving

$$y = 10(6) + 10 = 70$$

or \$700,000. ■

APPLIED EXAMPLE 11 Appreciation in Value of an Art Object Suppose an art object purchased for \$50,000 is expected to appreciate in value at a constant rate of \$5000 per year for the next 5 years. Use Equation (5) to write an equation predicting the value of the art object in the next several years. What will be its value 3 years from the purchase date?

Solution Let x denote the time (in years) that has elapsed since the purchase date and let y denote the object's value (in dollars). Then, $y = 50{,}000$ when $x = 0$. Furthermore, the slope of the required equation is given by $m = 5000$, since each unit increase in x (1 year) implies an increase of 5000 units (dollars) in y. Using Equation (5) with $m = 5000$ and $b = 50{,}000$, we obtain

$$y = 5000x + 50{,}000 \qquad y = mx + b$$

Three years from the purchase date, the value of the object will be given by

$$y = 5000(3) + 50{,}000$$

or \$65,000. ■

Explore & Discuss

Refer to Example 11. Can the equation predicting the value of the art object be used to predict long-term growth?

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 17

General Form of an Equation of a Line

We have considered several forms of the equation of a straight line in the plane. These different forms of the equation are equivalent to each other. In fact, each is a special case of the following equation.

General Form of a Linear Equation

The equation

$$Ax + By + C = 0 \tag{6}$$

where A, B, and C are constants and A and B are not both zero, is called the general form of a linear equation in the variables x and y.

We now state (without proof) an important result concerning the algebraic representation of straight lines in the plane.

THEOREM 1

An equation of a straight line is a linear equation; conversely, every linear equation represents a straight line.

This result justifies the use of the adjective *linear* in describing Equation (6).

EXAMPLE 12 Sketch the straight line represented by the equation

$$3x - 4y - 12 = 0$$

Solution Since every straight line is uniquely determined by two distinct points, we need to find only two points through which the line passes in order to sketch it. For convenience, let's compute the points at which the line crosses the x- and y-axes. Setting $y = 0$, we find $x = 4$, the x-intercept, so the line crosses the x-axis at the point (4, 0). Setting $x = 0$ gives $y = -3$, the y-intercept, so the line crosses the y-axis at the point $(0, -3)$. A sketch of the line appears in Figure 27.

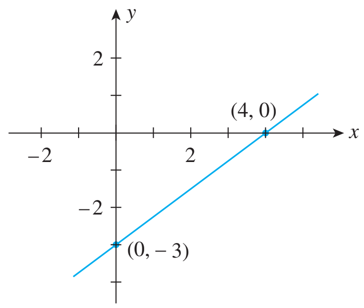


FIGURE 27
To sketch $3x - 4y - 12 = 0$, first find the x-intercept, 4, and the y-intercept, -3.

■

Here is a summary of the common forms of the equations of straight lines discussed in this section.

Equations of Straight Lines

Vertical line: $x = a$

Horizontal line: $y = b$

Point-slope form: $y - y_1 = m(x - x_1)$

Slope-intercept form: $y = mx + b$

General form: $Ax + By + C = 0$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 18

1.2 Self-Check Exercises

1. Determine the number a such that the line passing through the points $(a, 2)$ and $(3, 6)$ is parallel to a line with slope 4.

2. Find an equation of the line that passes through the point $(3, -1)$ and is perpendicular to a line with slope $-\frac{1}{2}$.

3. Does the point $(3, -3)$ lie on the line with equation $2x - 3y - 12 = 0$? Sketch the graph of the line.

4. **Satellite TV Subscribers** The following table gives the number of satellite TV subscribers in the United States (in millions) from 2004 through 2008 ($t = 0$ corresponds to the beginning of 2004):

Year, t	0	1	2	3	4
Number, y	22.5	24.8	27.1	29.1	30.7

   a. Plot the number of satellite TV subscribers in the United States (y) versus the year (t).
   b. Draw the line L through the points $(0, 22.5)$ and $(4, 30.7)$.
   c. Find an equation of the line L.
   d. Assuming that this trend continues, estimate the number of satellite TV subscribers in the United States in 2010.

   Sources: National Cable & Telecommunications Association and the Federal Communications Commission.

Solutions to Self-Check Exercises 1.2 can be found on page 23.

1.2 Concept Questions

1. What is the slope of a nonvertical line? What can you say about the slope of a vertical line?

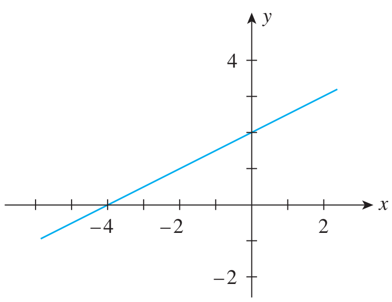
2. Give (a) the point-slope form, (b) the slope-intercept form, and (c) the general form of an equation of a line.

3. Let L_1 have slope m_1 and let L_2 have slope m_2. State the conditions on m_1 and m_2 if (a) L_1 is parallel to L_2 and (b) L_1 is perpendicular to L_2.

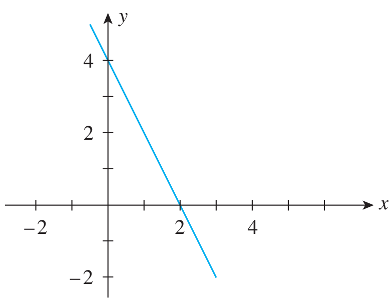
1.2 Exercises

In Exercises 1–4, find the slope of the line shown in each figure.

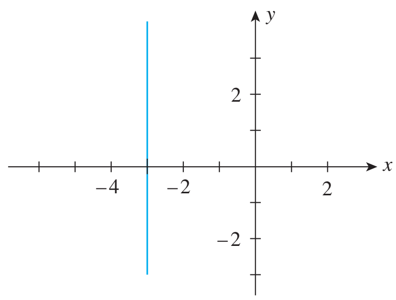
1.



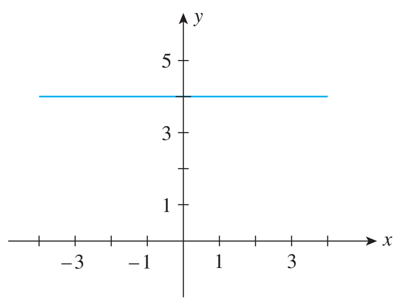
2.



3.



4.



48149_01_ch1_p001-066 11/2/10 10:35 AM Page 19

In Exercises 5–10, find the slope of the line that passes through the given pair of points.

5. (4, 3) and (5, 8)

6. (4, 5) and (3, 8)

7. (−2, 3) and (4, 8)

8. (−2, −2) and (4, −4)

9. (a, b) and (c, d)

10. $(-a + 1, b - 1)$ and $(a + 1, -b)$

11. Given the equation $y = 4x - 3$, answer the following questions.

a. If x increases by 1 unit, what is the corresponding change in y?

b. If x decreases by 2 units, what is the corresponding change in y?

12. Given the equation $2x + 3y = 4$, answer the following questions.

a. Is the slope of the line described by this equation positive or negative?

b. As x increases in value, does y increase or decrease?

c. If x decreases by 2 units, what is the corresponding change in y?

In Exercises 13 and 14, determine whether the lines through the pairs of points are parallel.

13. $A(1, -2)$, $B(-3, -10)$ and $C(1, 5)$, $D(-1, 1)$

14. $A(2, 3)$, $B(2, -2)$ and $C(-2, 4)$, $D(-2, 5)$

In Exercises 15 and 16, determine whether the lines through the pairs of points are perpendicular.

15. $A(-2, 5)$, $B(4, 2)$ and $C(-1, -2)$, $D(3, 6)$

16. $A(2, 0)$, $B(1, -2)$ and $C(4, 2)$, $D(-8, 4)$

17. If the line passing through the points $(1, a)$ and $(4, -2)$ is parallel to the line passing through the points (2, 8) and $(-7, a + 4)$, what is the value of a?

18. If the line passing through the points $(a, 1)$ and (5, 8) is parallel to the line passing through the points (4, 9) and $(a + 2, 1)$, what is the value of a?

19. Find an equation of the horizontal line that passes through (−4, −3).

20. Find an equation of the vertical line that passes through (0, 5).

In Exercises 21–26, match the statement with one of the graphs (a)–(f).

21. The slope of the line is zero.

22. The slope of the line is undefined.

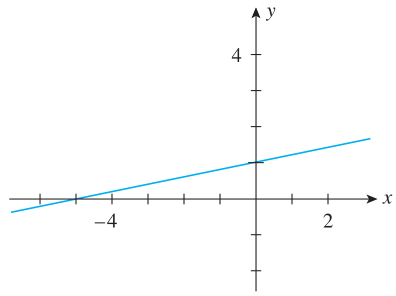
23. The slope of the line is positive, and its y-intercept is positive.

24. The slope of the line is positive, and its y-intercept is negative.

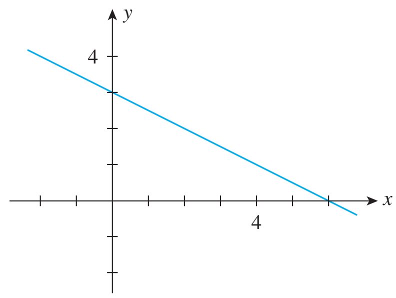
25. The slope of the line is negative, and its x-intercept is negative.

26. The slope of the line is negative, and its x-intercept is positive.

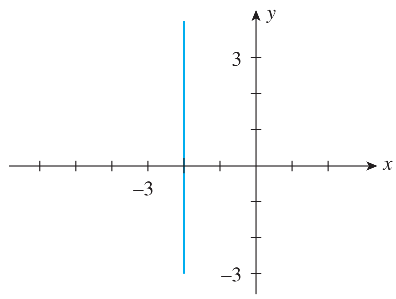
(a)



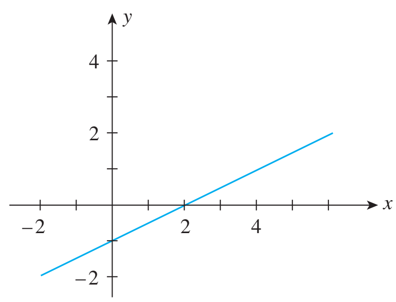
(b)



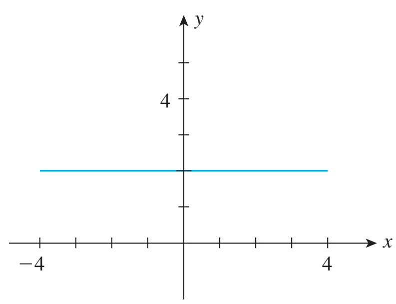
(c)



(d)

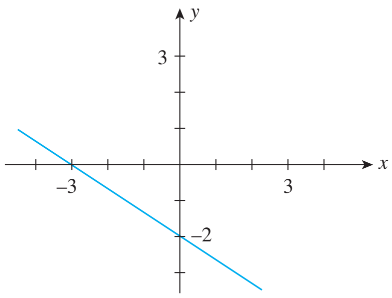


(e)



48149_01_ch1_p001-066 11/2/10 10:35 AM Page 20

(f)



In Exercises 27–30, find an equation of the line that passes through the point and has the indicated slope *m*.

27. $(3, -4);\ m = 2$

28. $(2, 4);\ m = -1$

29. $(-3, 2);\ m = 0$

30. $(1, 2);\ m = -\frac{1}{2}$

In Exercises 31–34, find an equation of the line that passes through the given points.

31. $(2, 4)$ and $(3, 7)$

32. $(2, 1)$ and $(2, 5)$

33. $(1, 2)$ and $(-3, -2)$

34. $(-1, -2)$ and $(3, -4)$

In Exercises 35–38, find an equation of the line that has slope *m* and *y*-intercept *b*.

35. $m = 3;\ b = 4$

36. $m = -2;\ b = -1$

37. $m = 0;\ b = 5$

38. $m = -\frac{1}{2};\ b = \frac{3}{4}$

In Exercises 39–44, write the equation in the slope-intercept form and then find the slope and *y*-intercept of the corresponding line.

39. $x - 2y = 0$

40. $y - 2 = 0$

41. $2x - 3y - 9 = 0$

42. $3x - 4y + 8 = 0$

43. $2x + 4y = 14$

44. $5x + 8y - 24 = 0$

45. Find an equation of the line that passes through the point $(-2, 2)$ and is parallel to the line $2x - 4y - 8 = 0$.

46. Find an equation of the line that passes through the point $(-1, 3)$ and is parallel to the line passing through the points $(-2, -3)$ and $(2, 5)$.

47. Find an equation of the line that passes through the point $(2, 4)$ and is perpendicular to the line $3x + 4y - 22 = 0$.

48. Find an equation of the line that passes through the point $(1, -2)$ and is perpendicular to the line passing through the points $(-2, -1)$ and $(4, 3)$.

In Exercises 49–54, find an equation of the line that satisfies the given condition.

49. The line parallel to the *x*-axis and 6 units below it

50. The line passing through the origin and parallel to the line passing through the points $(2, 4)$ and $(4, 7)$

51. The line passing through the point (a, b) with slope equal to zero

52. The line passing through $(-3, 4)$ and parallel to the *x*-axis

53. The line passing through $(-5, -4)$ and parallel to the line passing through $(-3, 2)$ and $(6, 8)$

54. The line passing through (a, b) with undefined slope

55. Given that the point $P(-3, 5)$ lies on the line $kx + 3y + 9 = 0$, find k.

56. Given that the point $P(2, -3)$ lies on the line $-2x + ky + 10 = 0$, find k.

In Exercises 57–62, sketch the straight line defined by the linear equation by finding the *x*- and *y*-intercepts.

Hint: See Example 12.

57. $3x - 2y + 6 = 0$

58. $2x - 5y + 10 = 0$

59. $x + 2y - 4 = 0$

60. $2x + 3y - 15 = 0$

61. $y + 5 = 0$

62. $-2x - 8y + 24 = 0$

63. Show that an equation of a line through the points $(a, 0)$ and $(0, b)$ with $a \neq 0$ and $b \neq 0$ can be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

(Recall that the numbers *a* and *b* are the *x*- and *y*-intercepts, respectively, of the line. This form of an equation of a line is called the **intercept form**.)

In Exercises 64–67, use the results of Exercise 63 to find an equation of a line with the *x*- and *y*-intercepts.

64. *x*-intercept 3; *y*-intercept 4

65. *x*-intercept -2; *y*-intercept -4

66. *x*-intercept $-\frac{1}{2}$; *y*-intercept $\frac{3}{4}$

67. *x*-intercept 4; *y*-intercept $-\frac{1}{2}$

In Exercises 68 and 69, determine whether the points lie on a straight line.

68. $A(-1, 7)$, $B(2, -2)$, and $C(5, -9)$

69. $A(-2, 1)$, $B(1, 7)$, and $C(4, 13)$

70. **Temperature Conversion** The relationship between the temperature in degrees Fahrenheit (°F) and the temperature in degrees Celsius (°C) is

$$F = \frac{9}{5}C + 32$$

a. Sketch the line with the given equation.
b. What is the slope of the line? What does it represent?
c. What is the *F*-intercept of the line? What does it represent?

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 21

71. Nuclear Plant Utilization The United States is not building many nuclear plants, but the ones it has are running at nearly full capacity. The output (as a percentage of total capacity) of nuclear plants is described by the equation

$$y = 1.9467t + 70.082$$

where t is measured in years, with $t = 0$ corresponding to the beginning of 1990.

a. Sketch the line with the given equation.
b. What are the slope and the y-intercept of the line found in part (a)?
c. Give an interpretation of the slope and the y-intercept of the line found in part (a).
d. If the utilization of nuclear power continued to grow at the same rate and the total capacity of nuclear plants in the United States remained constant, by what year were the plants generating at maximum capacity?

Source: Nuclear Energy Institute.

72. Social Security Contributions For wages less than the maximum taxable wage base, Social Security contributions (including those for Medicare) by employees are 7.65% of the employee's wages.

a. Find an equation that expresses the relationship between the wages earned (x) and the Social Security taxes paid (y) by an employee who earns less than the maximum taxable wage base.
b. For each additional dollar that an employee earns, by how much is his or her Social Security contribution increased? (Assume that the employee's wages are less than the maximum taxable wage base.)
c. What Social Security contributions will an employee who earns \$65,000 (which is less than the maximum taxable wage base) be required to make?

Source: Social Security Administration.

73. College Admissions Using data compiled by the Admissions Office at Faber University, college admissions officers estimate that 55% of the students who are offered admission to the freshman class at the university will actually enroll.

a. Find an equation that expresses the relationship between the number of students who actually enroll (y) and the number of students who are offered admission to the university (x).
b. If the desired freshman class size for the upcoming academic year is 1100 students, how many students should be admitted?

74. Weight of Whales The equation $W = 3.51L - 192$, expressing the relationship between the length L (in feet) and the expected weight W (in British tons) of adult blue whales, was adopted in the late 1960s by the International Whaling Commission.

a. What is the expected weight of an 80-ft blue whale?
b. Sketch the straight line that represents the equation.

75. The Narrowing Gender Gap Since the founding of the Equal Employment Opportunity Commission and the passage of equal-pay laws, the gulf between men's and women's earnings has continued to close gradually. At the beginning of 1990 ($t = 0$), women's wages were 68% of men's wages, and by the beginning of 2000 ($t = 10$), women's wages were 80% of men's wages. If this gap between women's and men's wages continued to narrow *linearly*, then women's wages were what percentage of men's wages at the beginning of 2004?

Source: Journal of Economic Perspectives.

76. Sales of Navigation Systems The projected number of navigation systems (in millions) installed in vehicles in North America, Europe, and Japan from 2002 through 2006 are shown in the following table ($x = 0$ corresponds to 2002):

Year, x	0	1	2	3	4
Systems Installed, y	3.9	4.7	5.8	6.8	7.8

a. Plot the annual sales (y) versus the year (x).
b. Draw a straight line L through the points corresponding to 2002 and 2006.
c. Derive an equation of the line L.
d. Use the equation found in part (c) to estimate the number of navigation systems installed in 2005. Compare this figure with the sales for that year.

Source: ABI Research.

77. Sales of GPS Equipment The annual sales (in billions of dollars) of global positioning systems (GPS) equipment from 2000 through 2006 are shown in the following table ($x = 0$ corresponds to 2000):

Year, x	0	1	2	3	4	5	6
Annual Sales, y	7.9	9.6	11.5	13.3	15.2	17	18.8

a. Plot the annual sales (y) versus the year (x).
b. Draw a straight line L through the points corresponding to 2000 and 2006.
c. Derive an equation of the line L.
d. Use the equation found in part (c) to estimate the annual sales of GPS equipment in 2005. Compare this figure with the projected sales for that year.

Source: ABI Research.

78. Ideal Heights and Weights for Women The Venus Health Club for Women provides its members with the following table, which gives the average desirable weight (in pounds) for women of a given height (in inches):

Height, x	60	63	66	69	72
Weight, y	108	118	129	140	152

a. Plot the weight (y) versus the height (x).
b. Draw a straight line L through the points corresponding to heights of 5 ft and 6 ft.
c. Derive an equation of the line L.
d. Using the equation of part (c), estimate the average desirable weight for a woman who is 5 ft, 5 in. tall.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 22

79. **COST OF A COMMODITY** A manufacturer obtained the following data relating the cost y (in dollars) to the number of units (x) of a commodity produced:

Units Produced, x	0	20	40	60	80	100
Cost in Dollars, y	200	208	222	230	242	250

a. Plot the cost (y) versus the quantity produced (x).
b. Draw a straight line through the points (0, 200) and (100, 250).
c. Derive an equation of the straight line of part (b).
d. Taking this equation to be an approximation of the relationship between the cost and the level of production, estimate the cost of producing 54 units of the commodity.

80. **DIGITAL TV SERVICES** The percentage of homes with digital TV services stood at 5% at the beginning of 1999 ($t = 0$) and was projected to grow linearly so that, at the beginning of 2003 ($t = 4$), the percentage of such homes was 25%.
a. Derive an equation of the line passing through the points $A(0, 5)$ and $B(4, 25)$.
b. Plot the line with the equation found in part (a).
c. Using the equation found in part (a), find the percentage of homes with digital TV services at the beginning of 2001.

Source: Paul Kagan Associates.

81. **SALES GROWTH** Metro Department Store's annual sales (in millions of dollars) during the past 5 years were

Annual Sales, y	5.8	6.2	7.2	8.4	9.0
Year, x	1	2	3	4	5

a. Plot the annual sales (y) versus the year (x).
b. Draw a straight line L through the points corresponding to the first and fifth years.
c. Derive an equation of the line L.
d. Using the equation found in part (c), estimate Metro's annual sales 4 years from now ($x = 9$).

82. Is there a difference between the statements "The slope of a straight line is zero" and "The slope of a straight line does not exist (is not defined)"? Explain your answer.

83. Consider the slope-intercept form of a straight line $y = mx + b$. Describe the family of straight lines obtained by keeping
a. the value of m fixed and allowing the value of b to vary.
b. the value of b fixed and allowing the value of m to vary.

In Exercises 84–90, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

84. Suppose the slope of a line L is $-\frac{1}{2}$ and P is a given point on L. If Q is the point on L lying 4 units to the left of P, then Q is situated 2 units above P.

85. The point $(-1, 1)$ lies on the line with equation $3x + 7y = 5$.

86. The point $(1, k)$ lies on the line with equation $3x + 4y = 12$ if and only if $k = \frac{9}{4}$.

87. The line with equation $Ax + By + C = 0$ $(B \neq 0)$ and the line with equation $ax + by + c = 0$ $(b \neq 0)$ are parallel if $Ab - aB = 0$.

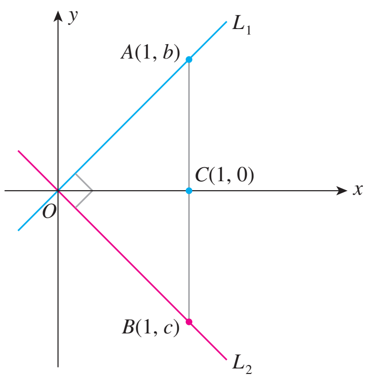
88. If the slope of the line L_1 is positive, then the slope of a line L_2 perpendicular to L_1 may be positive or negative.

89. The lines with equations $ax + by + c_1 = 0$ and $bx - ay + c_2 = 0$, where $a \neq 0$ and $b \neq 0$, are perpendicular to each other.

90. If L is the line with equation $Ax + By + C = 0$, where $A \neq 0$, then L crosses the x-axis at the point $(-C/A, 0)$.

91. Show that two distinct lines with equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, respectively, are parallel if and only if $a_1b_2 - b_1a_2 = 0$.
Hint: Write each equation in the slope-intercept form and compare.

92. Prove that if a line L_1 with slope m_1 is perpendicular to a line L_2 with slope m_2, then $m_1m_2 = -1$.
Hint: Refer to the accompanying figure. Show that $m_1 = b$ and $m_2 = c$. Next, apply the Pythagorean Theorem and the distance formula to the triangles OAC, OCB, and OBA to show that $1 = -bc$.



1.2 Solutions to Self-Check Exercises

1. The slope of the line that passes through the points $(a, 2)$ and $(3, 6)$ is

$$m = \frac{6 - 2}{3 - a} = \frac{4}{3 - a}$$

Since this line is parallel to a line with slope 4, m must be equal to 4; that is,

$$\frac{4}{3 - a} = 4$$

or, upon multiplying both sides of the equation by $3 - a$,

$$\begin{aligned} 4 &= 4(3 - a) \\ 4 &= 12 - 4a \\ 4a &= 8 \\ a &= 2 \end{aligned}$$

2. Since the required line L is perpendicular to a line with slope $-\frac{1}{2}$, the slope of L is

$$m = -\frac{1}{-\frac{1}{2}} = 2$$

Next, using the point-slope form of the equation of a line, we have

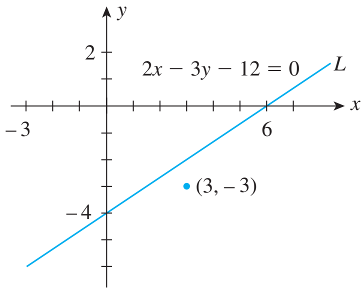
$$\begin{aligned} y - (-1) &= 2(x - 3) \\ y + 1 &= 2x - 6 \\ y &= 2x - 7 \end{aligned}$$

3. Substituting $x = 3$ and $y = -3$ into the left-hand side of the given equation, we find

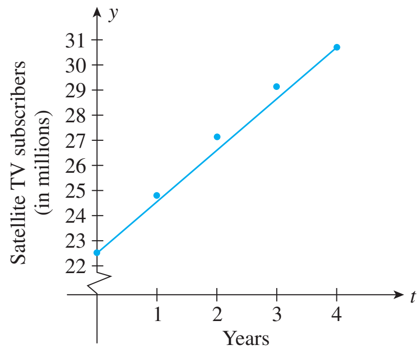
$$2(3) - 3(-3) - 12 = 3$$

which is not equal to zero (the right-hand side). Therefore, $(3, -3)$ does not lie on the line with equation $2x - 3y - 12 = 0$. (See the accompanying figure.)

Setting $x = 0$, we find $y = -4$, the y-intercept. Next, setting $y = 0$ gives $x = 6$, the x-intercept. We now draw the line passing through the points $(0, -4)$ and $(6, 0)$, as shown.



4. a. and **b.** See the following figure.



c. The slope of L is

$$m = \frac{30.7 - 22.5}{4 - 0} = 2.05$$

Using the point-slope form of the equation of a line with the point $(0, 22.5)$, we find

$$\begin{aligned} y - 22.5 &= 2.05(t - 0) \\ y &= 2.05t + 22.5 \end{aligned}$$

d. Here the year 2010 corresponds to $t = 6$, so the estimated number of satellite TV subscribers in the United States in 2010 is

$$y = 2.05(6) + 22.5 = 34.8$$

or 34.8 million.

USING TECHNOLOGY

Graphing a Straight Line

Graphing Utility

The first step in plotting a straight line with a graphing utility is to select a suitable viewing window. We usually do this by experimenting. For example, you might first plot the straight line using the **standard viewing window** $[-10, 10] \times [-10, 10]$. If necessary, you then might adjust the viewing window by enlarging it or reducing it to obtain a sufficiently complete view of the line or at least the portion of the line that is of interest.

(continued)

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 24

EXAMPLE 1 Plot the straight line $2x + 3y - 6 = 0$ in the standard viewing window.

Solution The straight line in the standard viewing window is shown in Figure T1.

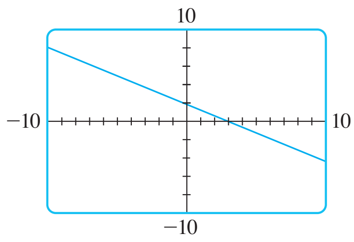
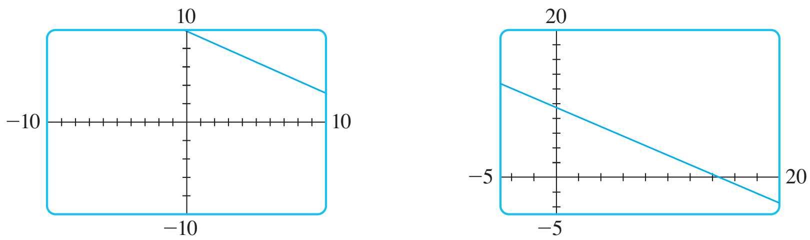


FIGURE **T1**
The straight line $2x + 3y - 6 = 0$ in the standard viewing window

EXAMPLE 2 Plot the straight line $2x + 3y - 30 = 0$ in (a) the standard viewing window and (b) the viewing window $[-5, 20] \times [-5, 20]$.

Solution

a. The straight line in the standard viewing window is shown in Figure T2a.
b. The straight line in the viewing window $[-5, 20] \times [-5, 20]$ is shown in Figure T2b. This figure certainly gives a more complete view of the straight line.



(a) The graph of $2x + 3y - 30 = 0$ in the standard viewing window

(b) The graph of $2x + 3y - 30 = 0$ in the viewing window $[-5, 20] \times [-5, 20]$

FIGURE **T2**

Excel

In the examples and exercises that follow, we assume that you are familiar with the basic features of Microsoft Excel. Please consult your Excel manual or use Excel's Help features to answer questions regarding the standard commands and operating instructions for Excel. Here we use Microsoft Excel 2010.*

EXAMPLE 3 Plot the graph of the straight line $2x + 3y - 6 = 0$ over the interval $[-10, 10]$.

Solution

1. *Write the equation in the slope-intercept form:*

$$y = -\frac{2}{3}x + 2$$

2. *Create a table of values.* First, enter the input values: Enter the values of the endpoints of the interval over which you are graphing the straight line. (Recall that we need only two distinct data points to draw the graph of a straight line. In general, we select the endpoints of the interval over which the straight line is to be drawn as our data points.) In this case, we enter −10 in cell B1 and 10 in cell C1.

Second, enter the formula for computing the y-values: Here, we enter

```
= −(2/3)*B1+2
```

in cell B2 and then press **Enter**.

*Instructions for solving these examples and exercises using Microsoft Excel 2007 are given on CourseMate.

Third, evaluate the function at the other input value: To extend the formula to cell C2, move the pointer to the small black box at the lower right corner of cell B2 (the cell containing the formula). Observe that the pointer now appears as a black + (plus sign). Drag this pointer through cell C2 and then release it. The *y*-value, – 4.66667, corresponding to the *x*-value in cell C1(10) will appear in cell C2 (Figure T3).

	A	B	C
1	x	-10	10
2	y	8.666667	-4.66667

FIGURE **T3**
Table of values for *x* and *y*

3. *Graph the straight line determined by these points.* First, highlight the numerical values in the table. Here we highlight cells B1:B2 and C1:C2.

Step 1 Click on the **Insert** ribbon tab and then select **Scatter** from the Charts group. Select the chart subtype in the first row and second column. A chart will then appear on your worksheet.

Step 2 From the Chart Tools group that now appears at the end of the ribbon, click the **Layout** tab and then select **Chart Title** from the Labels group followed by **Above Chart**. Type `y =-(2/3)x + 2` and press **Enter**. Click **Axis Titles** from the Labels group and select **Primary Horizontal Axis Title** followed by **Title Below Axis**. Type `x` and then press **Enter**. Next, click **Axis Titles** again and select **Primary Vertical Axis Title** followed by **Vertical Title**. Type `y` and press **Enter**.

Step 3 Click **Series1** which appears on the right side of the graph and press **Delete**.

The graph shown in Figure T4 will appear.

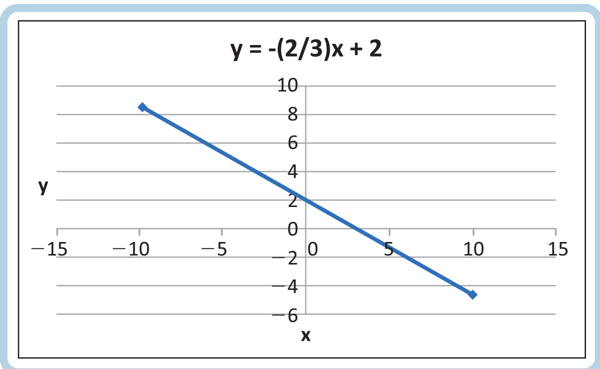


FIGURE **T4**
The graph of $y = -\frac{2}{3}x + 2$ over the interval $[-10, 10]$

If the interval over which the straight line is to be plotted is not specified, then you might have to experiment to find an appropriate interval for the *x*-values in your graph. For example, you might first plot the straight line over the interval [−10, 10]. If necessary you then might adjust the interval by enlarging it or reducing it to obtain a sufficiently complete view of the line or at least the portion of the line that is of interest.

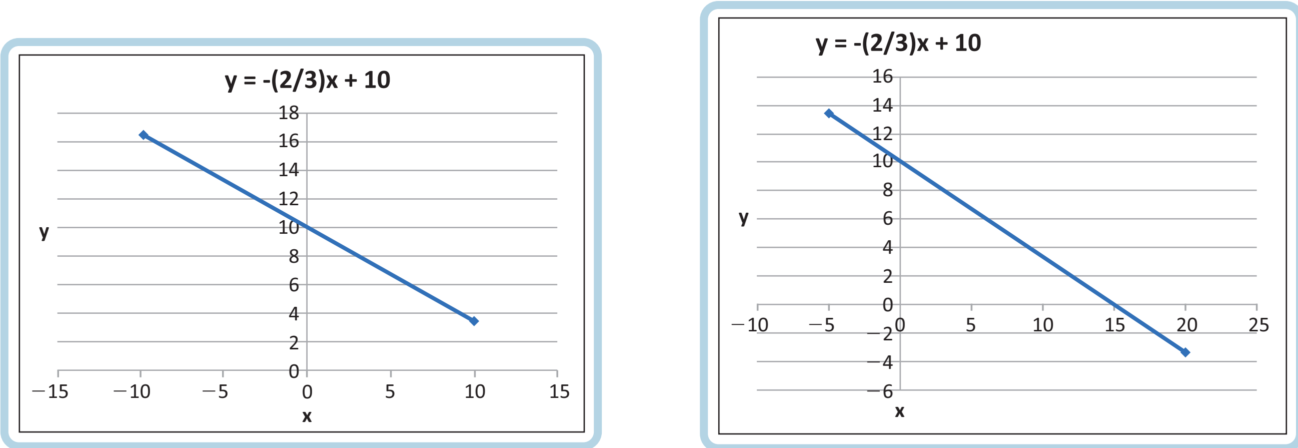
Note: Boldfaced words/characters enclosed in a box (for example, **Enter**) indicate that an action (click, select, or press) is required. Words/characters printed blue (for example, Chart Type) indicate words/characters that appear on the screen. Words/characters printed in a monospace font (for example, `=(−2/3)*A2+2`) indicate words/characters that need to be typed and entered.

(*continued*)

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 26

EXAMPLE 4 Plot the straight line $2x + 3y - 30 = 0$ over the intervals (a) $[-10, 10]$ and (b) $[-5, 20]$.

Solution **a** and **b.** We first cast the equation in the slope-intercept form, obtaining $y = -\frac{2}{3}x + 10$. Following the procedure given in Example 3, we obtain the graphs shown in Figure T5.



(a) (b)

FIGURE **T5**
The graph of $y = -\frac{2}{3}x + 10$ over the intervals (a) $[-10, 10]$ and (b) $[-5, 20]$

Observe that the graph in Figure T5b includes the x- and y-intercepts. This figure certainly gives a more complete view of the straight line. ■

TECHNOLOGY EXERCISES

Graphing Utility

In Exercises 1–4, plot the straight line with the equation in the standard viewing window.

1. $3.2x + 2.1y - 6.72 = 0$ **2.** $2.3x - 4.1y - 9.43 = 0$

3. $1.6x + 5.1y = 8.16$ **4.** $-3.2x + 2.1y = 6.72$

In Exercises 5–8, plot the straight line with the equation in (a) the standard viewing window and (b) the indicated viewing window.

5. $12.1x + 4.1y - 49.61 = 0$; $[-10, 10] \times [-10, 20]$

6. $4.1x - 15.2y - 62.32 = 0$; $[-10, 20] \times [-10, 10]$

7. $20x + 16y = 300$; $[-10, 20] \times [-10, 30]$

8. $32.2x + 21y = 676.2$; $[-10, 30] \times [-10, 40]$

In Exercises 9–12, plot the straight line with the equation in an appropriate viewing window. (*Note*: The answer is *not* unique.)

9. $20x + 30y = 600$

10. $30x - 20y = 600$

11. $22.4x + 16.1y - 352 = 0$

12. $18.2x - 15.1y = 274.8$

Excel

In Exercises 1–4, plot the straight line with the equation over the interval $[-10, 10]$.

1. $3.2x + 2.1y - 6.72 = 0$ **2.** $2.3x - 4.1y - 9.43 = 0$

3. $1.6x + 5.1y = 8.16$ **4.** $-3.2x + 2.1y = 6.72$

In Exercises 5–8, plot the straight line with the equation over the given interval.

5. $12.1x + 4.1y - 49.61 = 0$; $[-10, 10]$

6. $4.1x - 15.2y - 62.32 = 0$; $[-10, 20]$

7. $20x + 16y = 300$; $[-10, 20]$

8. $32.2x + 21y = 676.2$; $[-10, 30]$

In Exercises 9–12, plot the straight line with the equation. (*Note*: The answer is *not* unique.)

9. $20x + 30y = 600$

10. $30x - 20y = 600$

11. $22.4x + 16.1y - 352 = 0$

12. $18.2x - 15.1y = 274.8$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 27

1.3 Linear Functions and Mathematical Models

Mathematical Models

Regardless of the field from which a real-world problem is drawn, the problem is solved by analyzing it through a process called **mathematical modeling.** The four steps in this process, as illustrated in Figure 28, follow.

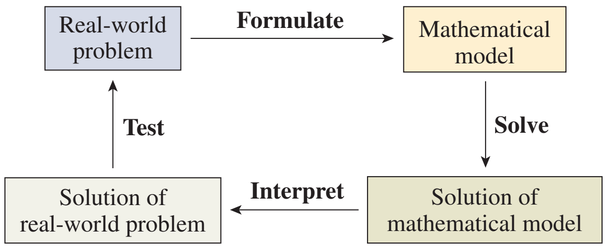


FIGURE **28**

Mathematical Modeling

1. **Formulate** Given a real-world problem, our first task is to formulate the problem using the language of mathematics. The many techniques that are used in constructing mathematical models range from theoretical consideration of the problem on the one extreme to an interpretation of data associated with the problem on the other. For example, the mathematical model giving the accumulated amount at any time when a certain sum of money is deposited in the bank can be derived theoretically (see Chapter 5). On the other hand, many of the mathematical models in this book are constructed by studying the data associated with the problem. In Section 1.5, we see how linear equations (models) can be constructed from a given set of data points. Also, in the ensuing chapters we will see how other mathematical models, including statistical and probability models, are used to describe and analyze real-world situations.
2. **Solve** Once a mathematical model has been constructed, we can use the appropriate mathematical techniques, which we will develop throughout the book, to solve the problem.
3. **Interpret** Bearing in mind that the solution obtained in Step 2 is just the solution of the mathematical model, we need to interpret these results in the context of the original real-world problem.
4. **Test** Some mathematical models of real-world applications describe the situations with complete accuracy. For example, the model describing a deposit in a bank account gives the exact accumulated amount in the account at any time. But other mathematical models give, at best, an approximate description of the real-world problem. In this case, we need to test the accuracy of the model by observing how well it describes the original real-world problem and how well it predicts past and/or future behavior. If the results are unsatisfactory, then we may have to reconsider the assumptions made in the construction of the model or, in the worst case, return to Step 1.

We now look at an important way of describing the relationship between two quantities using the notion of a function. As you will see subsequently, many mathematical models are represented by functions.

Functions

A manufacturer would like to know how his company's profit is related to its production level; a biologist would like to know how the population of a certain culture of bacteria will change with time; a psychologist would like to know the relationship between the learning time of an individual and the length of a vocabulary list; and a chemist would like to know how the initial speed of a chemical reaction is related to the amount of substrate used. In each instance, we are concerned with the same ques-

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 28

tion: How does one quantity depend on another? The relationship between two quantities is conveniently described in mathematics by using the concept of a function.

> **Function**
>
> A **function** f is a rule that assigns to each value of x one and only one value of y.

The number y is normally denoted by $f(x)$, read "f of x," emphasizing the dependency of y on x.

An example of a function may be drawn from the familiar relationship between the area of a circle and its radius. Let x and y denote the radius and area of a circle, respectively. From elementary geometry, we have

$$y = \pi x^2$$

This equation defines y as a function of x, since for each admissible value of x (a positive number representing the radius of a certain circle), there corresponds precisely one number $y = \pi x^2$ giving the area of the circle. This *area function* may be written as

$$f(x) = \pi x^2 \tag{7}$$

For example, to compute the area of a circle with a radius of 5 inches, we simply replace x in Equation (7) by the number 5. Thus, the area of the circle is

$$f(5) = \pi 5^2 = 25\pi$$

or 25π square inches.

Suppose we are given the function $y = f(x)$.* The variable x is referred to as the **independent variable,** and the variable y is called the **dependent variable.** The set of all values that may be assumed by x is called the **domain** of the function f, and the set comprising all the values assumed by $y = f(x)$ as x takes on all possible values in its domain is called the **range** of the function f. For the area function (7), the domain of f is the set of all positive numbers x, and the range of f is the set of all positive numbers y.

We now focus our attention on an important class of functions known as linear functions. Recall that a linear equation in x and y has the form $Ax + By + C = 0$, where A, B, and C are constants and A and B are not both zero. If $B \neq 0$, the equation can always be solved for y in terms of x; in fact, as we saw in Section 1.2, the equation may be cast in the slope-intercept form:

$$y = mx + b \qquad (m, b \text{ constants}) \tag{8}$$

Equation (8) defines y as a function of x. The domain and range of this function are the set of all real numbers. Furthermore, the graph of this function, as we saw in Section 1.2, is a straight line in the plane. For this reason, the function $f(x) = mx + b$ is called a linear function.

> **Linear Function**
>
> The function f defined by
>
> $$f(x) = mx + b$$
>
> where m and b are constants, is called a **linear function.**

Linear functions play an important role in the quantitative analysis of business and economic problems. First, many problems that arise in these and other fields are

*It is customary to refer to a function f as $f(x)$.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 29

linear in nature or are linear in the intervals of interest and thus can be formulated in terms of linear functions. Second, because linear functions are relatively easy to work with, assumptions involving linearity are often made in the formulation of problems. In many cases, these assumptions are justified, and acceptable mathematical models are obtained that approximate real-life situations.

The following example uses a linear function to model the market for U.S. health-care costs. In Section 1.5, we show how this model is constructed using the least-squares technique. (In "Using Technology" on pages 60–63, you will be asked to use a graphing calculator or Excel to construct other mathematical models from raw data.)

APPLIED EXAMPLE 1 U.S. Health-Care Expenditures Because the over-65 population will be growing more rapidly in the next few decades, health-care spending is expected to increase significantly in the coming decades. The following table gives the projected U.S. health-care expenditure (in trillions of dollars) from 2008 through 2013 (the figures after 2009 are estimates):

Year	2008	2009	2010	2011	2012	2013
Expenditure	2.34	2.47	2.57	2.70	2.85	3.02

A mathematical model giving the approximate U.S. health-care expenditures over the period in question is given by

$$S(t) = 0.134t + 2.325$$

where t is measured in years, with $t = 0$ corresponding to 2008.

a. Sketch the graph of the function S and the given data on the same set of axes.
b. Assuming that the trend continues, how much will U.S. health-care expenditures be in 2014 ($t = 6$)?
c. What is the projected rate of increase of U.S. health-care expenditures over the period in question?

Source: Centers for Medicare & Medicaid Services.

Solution

a. The graph of S is shown in Figure 29.

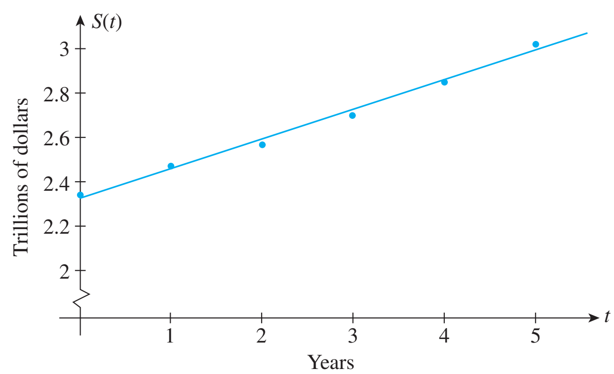


FIGURE **29**
Projected U.S. health-care expenditures from 2008 to 2013

b. The projected U.S. health-care expenditure in 2014 is

$$S(6) = 0.134(6) + 2.325 = 3.129$$

or approximately \$3.13 trillion.

c. The function S is linear; hence, we see that the rate of increase of the U.S. health-care expenditures is given by the slope of the straight line represented by S, which is approximately \$0.13 trillion per year. ■

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 30

In the rest of this section, we look at several applications that can be modeled by using linear functions.

Simple Depreciation

We first discussed linear depreciation in Section 1.2 as a real-world application of straight lines. The following example illustrates how to derive an equation describing the book value of an asset that is being depreciated linearly.

APPLIED EXAMPLE 2 Linear Depreciation A network server has an original value of \$10,000 and is to be depreciated linearly over 5 years with a \$3000 scrap value. Find an expression giving the book value at the end of year t. What will be the book value of the server at the end of the second year? What is the rate of depreciation of the server?

Solution Let $V(t)$ denote the network server's book value at the end of the tth year. Since the depreciation is linear, V is a linear function of t. Equivalently, the graph of the function is a straight line. Now, to find an equation of the straight line, observe that $V = 10{,}000$ when $t = 0$; this tells us that the line passes through the point (0, 10,000). Similarly, the condition that $V = 3000$ when $t = 5$ says that the line also passes through the point (5, 3000). The slope of the line is given by

$$m = \frac{10{,}000 - 3000}{0 - 5} = -\frac{7000}{5} = -1400$$

Using the point-slope form of the equation of a line with the point (0, 10,000) and the slope $m = -1400$, we have

$$V - 10{,}000 = -1400(t - 0)$$
$$V = -1400t + 10{,}000$$

the required expression. The book value at the end of the second year is given by

$$V(2) = -1400(2) + 10{,}000 = 7200$$

or \$7200. The rate of depreciation of the server is given by the negative of the slope of the depreciation line. Since the slope of the line is $m = -1400$, the rate of depreciation is \$1400 per year. The graph of $V = -1400t + 10{,}000$ is sketched in Figure 30. ■

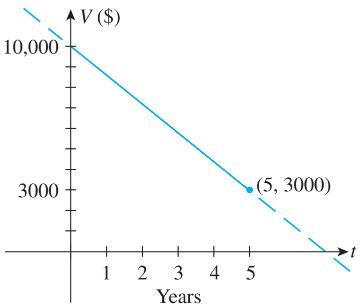


FIGURE 30
Linear depreciation of an asset

Linear Cost, Revenue, and Profit Functions

Whether a business is a sole proprietorship or a large corporation, the owner or chief executive must constantly keep track of operating costs, revenue resulting from the sale of products or services, and, perhaps most important, the profits realized. Three functions provide management with a measure of these quantities: the total cost function, the revenue function, and the profit function.

Cost, Revenue, and Profit Functions

Let x denote the number of units of a product manufactured or sold. Then, the **total cost function** is

$$C(x) = \text{Total cost of manufacturing } x \text{ units of the product}$$

The **revenue function** is

$$R(x) = \text{Total revenue realized from the sale of } x \text{ units of the product}$$

The **profit function** is

$$P(x) = \text{Total profit realized from manufacturing and selling } x \text{ units of the product}$$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 31

Generally speaking, the total cost, revenue, and profit functions associated with a company will probably be nonlinear (these functions are best studied using the tools of calculus). But *linear* cost, revenue, and profit functions do arise in practice, and we will consider such functions in this section. Before deriving explicit forms of these functions, we need to recall some common terminology.

The costs that are incurred in operating a business are usually classified into two categories. Costs that remain more or less constant regardless of the firm's activity level are called **fixed costs.** Examples of fixed costs are rental fees and executive salaries. Costs that vary with production or sales are called **variable costs.** Examples of variable costs are wages and costs for raw materials.

Suppose a firm has a fixed cost of F dollars, a production cost of c dollars per unit, and a selling price of s dollars per unit. Then the *cost function* $C(x)$, the *revenue function* $R(x)$, and the *profit function* $P(x)$ for the firm are given by

$$\begin{aligned} C(x) &= cx + F \\ R(x) &= sx \\ P(x) &= R(x) - C(x) \qquad \text{Revenue} - \text{cost} \\ &= (s - c)x - F \end{aligned}$$

where x denotes the number of units of the commodity produced and sold. The functions C, R, and P are linear functions of x.

APPLIED EXAMPLE 3 Profit Functions Puritron, a manufacturer of water filters, has a monthly fixed cost of \$20,000, a production cost of \$20 per unit, and a selling price of \$30 per unit. Find the cost function, the revenue function, and the profit function for Puritron.

Solution Let x denote the number of units produced and sold. Then

$$\begin{aligned} C(x) &= 20x + 20{,}000 \\ R(x) &= 30x \\ P(x) &= R(x) - C(x) \\ &= 30x - (20x + 20{,}000) \\ &= 10x - 20{,}000 \end{aligned}$$

■

Linear Demand and Supply Curves

In a free-market economy, consumer demand for a particular commodity depends on the commodity's unit price. A **demand equation** expresses this relationship between the unit price and the quantity demanded. The corresponding graph of the demand equation is called a **demand curve.** In general, the quantity demanded of a commodity decreases as its unit price increases, and vice versa. Accordingly, a **demand function** defined by $p = f(x)$, where p measures the unit price and x measures the number of units of the commodity, is generally characterized as a decreasing function of x; that is, $p = f(x)$ decreases as x increases.

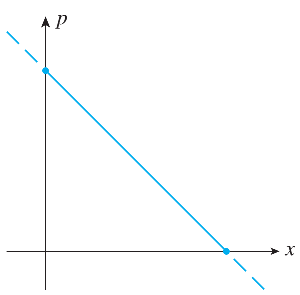


FIGURE 31
A graph of a linear demand function

The simplest demand function is defined by a linear equation in x and p, where both x and p assume only positive values. Its graph is a straight line having a negative slope. Thus, the demand curve in this case is that part of the graph of a straight line that lies in the first quadrant (Figure 31).

APPLIED EXAMPLE 4 Demand Functions The quantity demanded of the Sentinel iPod™ alarm clock is 48,000 units when the unit price is \$8. At \$12 per unit, the quantity demanded drops to 32,000 units. Find the demand equation, assuming that it is linear. What is the unit price corresponding to a

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 32

quantity demanded of 40,000 units? What is the quantity demanded if the unit price is \$14?

Solution Let p denote the unit price of an iPod alarm clock (in dollars) and let x (in units of 1000) denote the quantity demanded when the unit price of the clocks is \$$p$. If $p = 8$ then $x = 48$, and the point (48, 8) lies on the demand curve. Similarly, if $p = 12$, then $x = 32$, and the point (32, 12) also lies on the demand curve. Since the demand equation is linear, its graph is a straight line. The slope of the required line is given by

$$m = \frac{12 - 8}{32 - 48} = \frac{4}{-16} = -\frac{1}{4}$$

So, using the point-slope form of an equation of a line with the point (48, 8), we find that

$$p - 8 = -\frac{1}{4}(x - 48)$$
$$p = -\frac{1}{4}x + 20$$

is the required equation. The demand curve is shown in Figure 32.

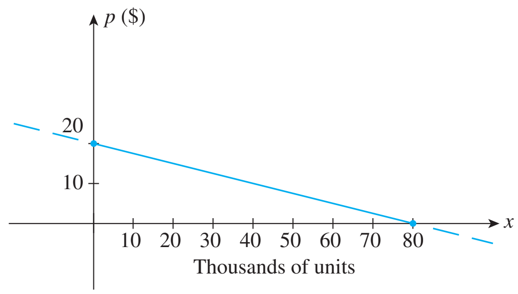


FIGURE 32
The graph of the demand equation $p = -\frac{1}{4}x + 20$

If the quantity demanded is 40,000 units ($x = 40$), the demand equation yields

$$y = -\frac{1}{4}(40) + 20 = 10$$

and we see that the corresponding unit price is \$10. Next, if the unit price is \$14 ($p = 14$), the demand equation yields

$$14 = -\frac{1}{4}x + 20$$
$$\frac{1}{4}x = 6$$
$$x = 24$$

so the quantity demanded will be 24,000 iPod alarm clocks. ■

In a competitive market, a relationship also exists between the unit price of a commodity and its availability in the market. In general, an increase in a commodity's unit price will induce the manufacturer to increase the supply of that commodity. Conversely, a decrease in the unit price generally leads to a drop in the supply. An equation that expresses the relationship between the unit price and the quantity supplied is called a **supply equation,** and the corresponding graph is called a **supply curve.** A **supply function,** defined by $p = f(x)$, is generally characterized by an increasing function of x; that is, $p = f(x)$ increases as x increases.

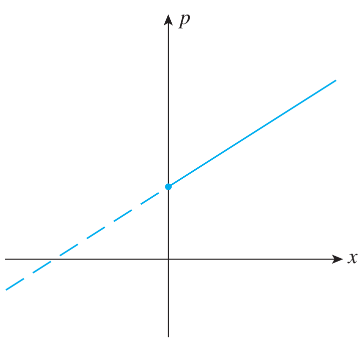


FIGURE 33
A graph of a linear supply function

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 33

As in the case of a demand equation, the simplest supply equation is a linear equation in p and x, where p and x have the same meaning as before but the line has a positive slope. The supply curve corresponding to a linear supply function is that part of the straight line that lies in the first quadrant (Figure 33).

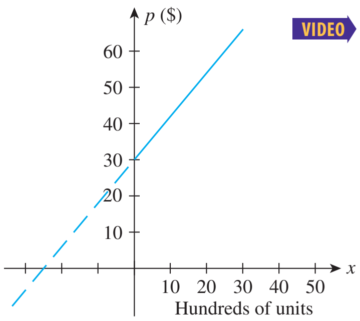


FIGURE **34**
The graph of the supply equation $4p - 5x = 120$

VIDEO

APPLIED EXAMPLE 5 Supply Functions The supply equation for a commodity is given by $4p - 5x = 120$, where p is measured in dollars and x is measured in units of 100.

a. Sketch the corresponding curve.
b. How many units will be marketed when the unit price is \$55?

Solution

a. Setting $x = 0$, we find the p-intercept to be 30. Next, setting $p = 0$, we find the x-intercept to be -24. The supply curve is sketched in Figure 34.
b. Substituting $p = 55$ in the supply equation, we have $4(55) - 5x = 120$ or $x = 20$, so the amount marketed will be 2000 units.

1.3 Self-Check Exercises

1. A manufacturer has a monthly fixed cost of \$60,000 and a production cost of \$10 for each unit produced. The product sells for \$15/unit.
a. What is the cost function?
b. What is the revenue function?
c. What is the profit function?
d. Compute the profit (loss) corresponding to production levels of 10,000 and 14,000 units/month.

2. The quantity demanded for a certain make of 30-in. × 52-in. area rug is 500 when the unit price is \$100. For each \$20 decrease in the unit price, the quantity demanded increases by 500 units. Find the demand equation and sketch its graph.

Solutions to Self-Check Exercises 1.3 can be found on page 36.

1.3 Concept Questions

1. a. What is a *function*? Give an example.
b. What is a *linear function*? Give an example.
c. What is the domain of a linear function? The range?
d. What is the graph of a linear function?

2. What is the general form of a linear cost function? A linear revenue function? A linear profit function?

3. Is the slope of a linear demand curve positive or negative? The slope of a linear supply curve?

1.3 Exercises

In Exercises 1–10, determine whether the equation defines y as a linear function of x. If so, write it in the form $y = mx + b$.

1. $2x + 3y = 6$
2. $-2x + 4y = 7$
3. $x = 2y - 4$
4. $2x = 3y + 8$
5. $2x - 4y + 9 = 0$
6. $3x - 6y + 7 = 0$
7. $2x^2 - 8y + 4 = 0$
8. $3\sqrt{x} + 4y = 0$
9. $2x - 3y^2 + 8 = 0$
10. $2x + \sqrt{y} - 4 = 0$

11. A manufacturer has a monthly fixed cost of \$40,000 and a production cost of \$8 for each unit produced. The product sells for \$12/unit.
a. What is the cost function?
b. What is the revenue function?
c. What is the profit function?
d. Compute the profit (loss) corresponding to production levels of 8000 and 12,000 units.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 34

12. A manufacturer has a monthly fixed cost of \$100,000 and a production cost of \$14 for each unit produced. The product sells for \$20/unit.
a. What is the cost function?
b. What is the revenue function?
c. What is the profit function?
d. Compute the profit (loss) corresponding to production levels of 12,000 and 20,000 units.

13. Find the constants m and b in the linear function $f(x) = mx + b$ such that $f(0) = 2$ and $f(3) = -1$.

14. Find the constants m and b in the linear function $f(x) = mx + b$ such that $f(2) = 4$ and the straight line represented by f has slope -1.

15. LINEAR DEPRECIATION An office building worth \$1 million when completed in 2005 is being depreciated linearly over 50 years. What was the book value of the building in 2010? What will it be in 2015? (Assume that the scrap value is \$0.)

16. LINEAR DEPRECIATION An automobile purchased for use by the manager of a firm at a price of \$24,000 is to be depreciated using the straight-line method over 5 years. What will be the book value of the automobile at the end of 3 years? (Assume that the scrap value is \$0.)

17. CONSUMPTION FUNCTIONS A certain economy's consumption function is given by the equation

$$C(x) = 0.75x + 6$$

where $C(x)$ is the personal consumption expenditure in billions of dollars and x is the disposable personal income in billions of dollars. Find $C(0)$, $C(50)$, and $C(100)$.

18. SALES TAX In a certain state, the sales tax T on the amount of taxable goods is 6% of the value of the goods purchased (x), where both T and x are measured in dollars.
a. Express T as a function of x.
b. Find $T(200)$ and $T(5.60)$.

19. SOCIAL SECURITY BENEFITS Social Security recipients receive an automatic cost-of-living adjustment (COLA) once each year. Their monthly benefit is increased by the same percentage that consumer prices have increased during the preceding year. Suppose consumer prices have increased by 5.3% during the preceding year.
a. Express the adjusted monthly benefit of a Social Security recipient as a function of his or her current monthly benefit.
b. If Carlos Garcia's monthly Social Security benefit is now \$1020, what will be his adjusted monthly benefit?

20. PROFIT FUNCTIONS AutoTime, a manufacturer of 24-hr variable timers, has a monthly fixed cost of \$48,000 and a production cost of \$8 for each timer manufactured. The timers sell for \$14 each.
a. What is the cost function?
b. What is the revenue function?
c. What is the profit function?
d. Compute the profit (loss) corresponding to production levels of 4000, 6000, and 10,000 timers, respectively.

21. PROFIT FUNCTIONS The management of TMI finds that the monthly fixed costs attributable to the production of their 100-watt light bulbs is \$12,100.00. If the cost of producing each twin-pack of light bulbs is \$0.60 and each twin-pack sells for \$1.15, find the company's cost function, revenue function, and profit function.

22. LINEAR DEPRECIATION In 2007, National Textile installed a new machine in one of its factories at a cost of \$250,000. The machine is depreciated linearly over 10 years with a scrap value of \$10,000.
a. Find an expression for the machine's book value in the tth year of use $(0 \leq t \leq 10)$.
b. Sketch the graph of the function of part (a).
c. Find the machine's book value in 2011.
d. Find the rate at which the machine is being depreciated.

23. LINEAR DEPRECIATION A workcenter system purchased at a cost of \$60,000 in 2010 has a scrap value of \$12,000 at the end of 4 years. If the straight-line method of depreciation is used,
a. Find the rate of depreciation.
b. Find the linear equation expressing the system's book value at the end of t years.
c. Sketch the graph of the function of part (b).
d. Find the system's book value at the end of the third year.

24. LINEAR DEPRECIATION Suppose an asset has an original value of \$$C$ and is depreciated linearly over N years with a scrap value of \$$S$. Show that the asset's book value at the end of the tth year is described by the function

$$V(t) = C - \left(\frac{C - S}{N}\right)t$$

Hint: Find an equation of the straight line passing through the points $(0, C)$ and (N, S). (Why?)

25. Rework Exercise 15 using the formula derived in Exercise 24.

26. Rework Exercise 16 using the formula derived in Exercise 24.

27. DRUG DOSAGES A method sometimes used by pediatricians to calculate the dosage of medicine for children is based on the child's surface area. If a denotes the adult dosage (in milligrams) and if S is the child's surface area (in square meters), then the child's dosage is given by

$$D(S) = \frac{Sa}{1.7}$$

a. Show that D is a linear function of S.
Hint: Think of D as having the form $D(S) = mS + b$. What are the slope m and the y-intercept b?
b. If the adult dose of a drug is 500 mg, how much should a child whose surface area is 0.4 m^2 receive?

28. DRUG DOSAGES Cowling's Rule is a method for calculating pediatric drug dosages. If a denotes the adult dosage (in milligrams) and if t is the child's age (in years), then the

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 35

child's dosage is given by

$$D(t) = \left(\frac{t+1}{24}\right)a$$

a. Show that D is a linear function of t.
Hint: Think of $D(t)$ as having the form $D(t) = mt + b$. What is the slope m and the y-intercept b?

b. If the adult dose of a drug is 500 mg, how much should a 4-year-old child receive?

29. **Broadband Internet Households** The number of U.S. broadband Internet households stood at 20 million at the beginning of 2002 and was projected to grow at the rate of 6.5 million households per year for the next 8 years.

a. Find a linear function $f(t)$ giving the projected number of U.S. broadband Internet households (in millions) in year t, where $t = 0$ corresponds to the beginning of 2002.

b. What was the projected number of U.S. broadband Internet households at the beginning of 2010?

Source: Jupiter Research.

30. **Dial-Up Internet Households** The number of U.S. dial-up Internet households stood at 42.5 million at the beginning of 2004 and was projected to decline at the rate of 3.9 million households per year for the next 6 years.

a. Find a linear function f giving the projected U.S. dial-up Internet households (in millions) in year t, where $t = 0$ corresponds to the beginning of 2004.

b. What was the projected number of U.S. dial-up Internet households at the beginning of 2010?

Source: Strategy Analytics, Inc.

31. **Celsius and Fahrenheit Temperatures** The relationship between temperature measured on the Celsius scale and on the Fahrenheit scale is linear. The freezing point is 0°C and 32°F, and the boiling point is 100°C and 212°F.

a. Find an equation giving the relationship between the temperature F measured on the Fahrenheit scale and the temperature C measured on the Celsius scale.

b. Find F as a function of C and use this formula to determine the temperature in Fahrenheit corresponding to a temperature of 20°C.

c. Find C as a function of F and use this formula to determine the temperature in Celsius corresponding to a temperature of 70°F.

32. **Cricket Chirping and Temperature** Entomologists have discovered that a linear relationship exists between the rate of chirping of crickets of a certain species and the air temperature. When the temperature is 70°F, the crickets chirp at the rate of 120 chirps/min, and when the temperature is 80°F, they chirp at the rate of 160 chirps/min.

a. Find an equation giving the relationship between the air temperature T and the number of chirps per minute N of the crickets.

b. Find N as a function of T, and use this function to determine the rate at which the crickets chirp when the temperature is 102°F.

For each demand equation in Exercises 33–36, where x represents the quantity demanded in units of 1000 and p is the unit price in dollars, (a) sketch the demand curve, and (b) determine the quantity demanded corresponding to the given unit price p.

33. $2x + 3p - 18 = 0;\ p = 4$

34. $5p + 4x - 80 = 0;\ p = 10$

35. $p = -3x + 60;\ p = 30$

36. $p = -0.4x + 120;\ p = 80$

37. **Demand Functions** At a unit price of \$55, the quantity demanded of a certain commodity is 1000 units. At a unit price of \$85, the demand drops to 600 units. Given that it is linear, find the demand equation. Above what price will there be no demand? What quantity would be demanded if the commodity were free?

38. **Demand Functions** The quantity demanded for a certain brand of portable CD players is 200 units when the unit price is set at \$90. The quantity demanded is 1200 units when the unit price is \$40. Find the demand equation, and sketch its graph.

39. **Demand Functions** Assume that a certain commodity's demand equation has the form $p = ax + b$, where x is the quantity demanded and p is the unit price in dollars. Suppose the quantity demanded is 1000 units when the unit price is \$9.00 and 6000 when the unit price is \$4.00. What is the quantity demanded when the unit price is \$7.50?

40. **Demand Functions** The demand equation for the Sicard sports watch is

$$p = -0.025x + 50$$

where x is the quantity demanded per week and p is the unit price in dollars. Sketch the graph of the demand equation. What is the highest price (theoretically) anyone would pay for the watch?

For each supply equation in Exercises 41–44, where x is the quantity supplied in units of 1000 and p is the unit price in dollars, (a) sketch the supply curve, and (b) determine the number of units of the commodity the supplier will make available in the market at the given unit price.

41. $3x - 4p + 24 = 0;\ p = 8$

42. $\frac{1}{2}x - \frac{2}{3}p + 12 = 0;\ p = 24$

43. $p = 2x + 10;\ p = 14$

44. $p = \frac{1}{2}x + 20;\ p = 28$

45. **Supply Functions** Suppliers of a certain brand of digital voice recorders will make 10,000 available in the market if the unit price is \$45. At a unit price of \$50, 20,000 units will be made available. Assuming that the relationship between the unit price and the quantity supplied is linear,

48149_01_ch1_p001-066 11/3/10 11:36 AM Page 36

derive the supply equation. Sketch the supply curve, and determine the quantity suppliers will make available when the unit price is \$70.

46. **Supply Functions** Producers will make 2000 refrigerators available when the unit price is \$330. At a unit price of \$390, 6000 refrigerators will be marketed. Find the equation relating the unit price of a refrigerator to the quantity supplied if the equation is known to be linear. How many refrigerators will be marketed when the unit price is \$450? What is the lowest price at which a refrigerator will be marketed?

In Exercises 47 and 48, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

47. Suppose $C(x) = cx + F$ and $R(x) = sx$ are the cost and revenue functions of a certain firm. Then the firm is making a profit if its level of production is less than $F/(s - c)$.

48. If $p = mx + b$ is a linear demand curve, then it is generally true that $m < 0$.

1.3 Solutions to Self-Check Exercises

1. Let x denote the number of units produced and sold. Then
a. $C(x) = 10x + 60{,}000$
b. $R(x) = 15x$
c. $P(x) = R(x) - C(x) = 15x - (10x + 60{,}000)$
$= 5x - 60{,}000$
d. $P(10{,}000) = 5(10{,}000) - 60{,}000$
$= -10{,}000$
or a loss of \$10,000 per month.
$P(14{,}000) = 5(14{,}000) - 60{,}000$
$= 10{,}000$
or a profit of \$10,000 per month.

2. Let p denote the price of a rug (in dollars), and let x denote the quantity of rugs demanded when the unit price is \$$p$. The condition that the quantity demanded is 500 when the unit price is \$100 tells us that the demand curve passes through the point (500, 100). Next, the condition that for each \$20 decrease in the unit price, the quantity demanded increases by 500 tells us that the demand curve is linear and that its slope is given by $-\frac{20}{500}$, or $-\frac{1}{25}$. Therefore, letting $m = -\frac{1}{25}$ in the demand equation

$$p = mx + b$$

we find

$$p = -\frac{1}{25}x + b$$

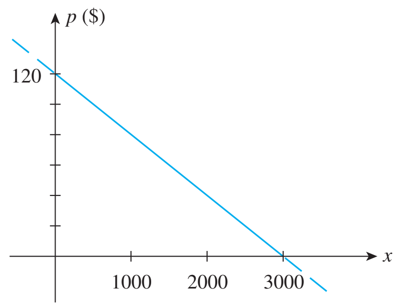
To determine b, use the fact that the straight line passes through (500, 100) to obtain

$$100 = -\frac{1}{25}(500) + b$$

or $b = 120$. Therefore, the required equation is

$$p = -\frac{1}{25}x + 120$$

The graph of the demand curve $p = -\frac{1}{25}x + 120$ is sketched in the following figure.



USING TECHNOLOGY

Evaluating a Function

Graphing Utility

A graphing utility can be used to find the value of a function f at a given point with minimal effort. However, to find the value of y for a given value of x in a linear equation such as $Ax + By + C = 0$, the equation must first be cast in the slope-intercept form $y = mx + b$, thus revealing the desired rule $f(x) = mx + b$ for y as a function of x.

EXAMPLE 1 Consider the equation $2x + 5y = 7$.

a. Plot the straight line with the given equation in the standard viewing window.
b. Find the value of y when $x = 2$ and verify your result by direct computation.
c. Find the value of y when $x = 1.732$.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 37

Solution

a. The straight line with equation $2x + 5y = 7$ or, equivalently, $y = -\frac{2}{5}x + \frac{7}{5}$ in the standard viewing window is shown in Figure T1.

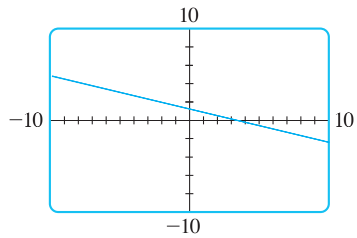


FIGURE **T1**
The straight line $2x + 5y = 7$ in the standard viewing window

b. Using the evaluation function of the graphing utility and the value of 2 for x, we find $y = 0.6$. This result is verified by computing

$$y = -\frac{2}{5}(2) + \frac{7}{5} = -\frac{4}{5} + \frac{7}{5} = \frac{3}{5} = 0.6$$

when $x = 2$.

c. Once again using the evaluation function of the graphing utility, this time with the value 1.732 for x, we find $y = 0.7072$. ■

When evaluating $f(x)$ at $x = a$, remember that the number a must lie between xMin and xMax.

APPLIED EXAMPLE 2 Market for Cholesterol-Reducing Drugs In a study conducted in early 2000, experts projected a rise in the market for cholesterol-reducing drugs. The U.S. market (in billions of dollars) for such drugs from 1999 through 2004 is approximated by

$$M(t) = 1.95t + 12.19$$

where t is measured in years, with $t = 0$ corresponding to 1999.

a. Plot the graph of the function M in the viewing window $[0, 5] \times [0, 25]$.
b. What was the estimated market for cholesterol-reducing drugs in 2005 ($t = 6$)?
c. What was the rate of increase of the market for cholesterol-reducing drugs over the period in question?

Source: S. G. Cowen.

Solution

a. The graph of M is shown in Figure T2.

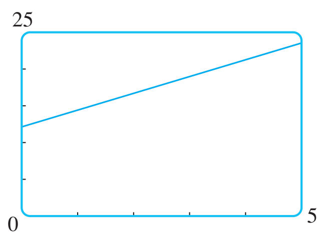


FIGURE **T2**
The graph of M in the viewing window $[0, 5] \times [0, 25]$

b. The market in 2005 for cholesterol-reducing drugs was approximately

$$M(6) = 1.95(6) + 12.19 = 23.89$$

or \$23.89 billion.

(continued)

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 38

c. The function M is linear; hence, we see that the rate of increase of the market for cholesterol-reducing drugs is given by the slope of the straight line represented by M, which is approximately \$1.95 billion per year. ■

Excel

Excel can be used to find the value of a function at a given value with minimal effort. However, to find the value of y for a given value of x in a linear equation such as $Ax + By + C = 0$, the equation must first be cast in the slope-intercept form $y = mx + b$, thus revealing the desired rule $f(x) = mx + b$ for y as a function of x.

EXAMPLE 3 Consider the equation $2x + 5y = 7$.

a. Find the value of y for $x = 0$, 5, and 10.
b. Plot the straight line with the given equation over the interval [0, 10].

Solution

a. Since this is a linear equation, we first cast the equation in slope-intercept form:

$$y = -\frac{2}{5}x + \frac{7}{5}$$

Next, we create a table of values (Figure T3), following the same procedure outlined in Example 3, pages 24–25. In this case we use the formula `=(-2/5)*B1+7/5` for the y-values.

	A	B	C	D
1	x	0	5	10
2	y	1.4	-0.6	-2.6

FIGURE **T3**
Table of values for x and y

b. Following the procedure outlined in Example 3, we obtain the graph shown in Figure T4.

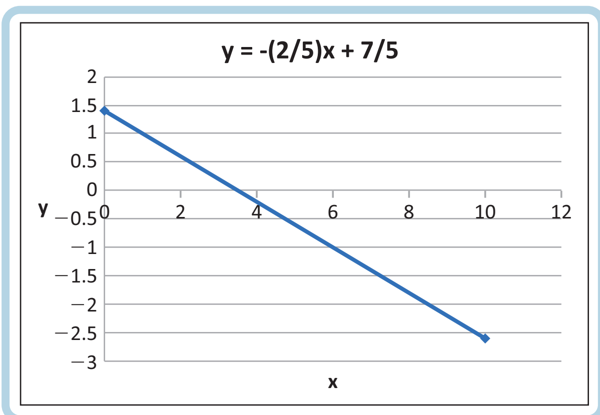


FIGURE **T4**
The graph of $y = -\frac{2}{5}x + \frac{7}{5}$ over the interval [0, 10] ■

Note: Words/characters printed in a monospace font (for example, `=(-2/3)*A2+2)` indicate words/characters that need to be typed and entered.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 39

APPLIED EXAMPLE 4 Market for Cholesterol-Reducing Drugs In a study conducted in early 2000, experts projected a rise in the market for cholesterol-reducing drugs. The U.S. market (in billions of dollars) for such drugs from 1999 through 2004 is approximated by

$$M(t) = 1.95t + 12.19$$

where t is measured in years, with $t = 0$ corresponding to 1999.

a. Plot the graph of the function M over the interval [0, 6].
b. What was the estimated market for cholesterol-reducing drugs in 2005 ($t = 6$)?
c. What was the rate of increase of the market for cholesterol-reducing drugs over the period in question?

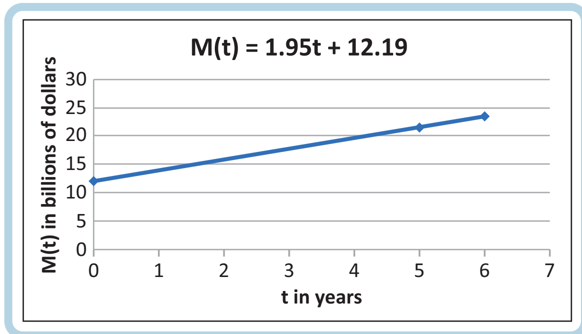
Source: S. G. Cowen.

Solution

a. Following the instructions given in Example 3, pages 24–26, we obtain the spreadsheet and graph shown in Figure T5. [*Note:* We have made the appropriate entries for the title and x- and y-axis labels. In particular, for Primary Vertical Axis Title, select **Rotated Title** and type `M(t) in billions of dollars`].

	A	B	C	D
1	t	0	5	6
2	M(t)	12.19	21.94	23.89

(a)



(b)

FIGURE **T5**
(a) The table of values for t and $M(t)$ and (b) the graph showing the demand for cholesterol-reducing drugs

b. From the table of values, we see that

$$M(6) = 1.95(6) + 12.19 = 23.89$$

or \$23.89 billion.

c. The function M is linear; hence, we see that the rate of increase of the market for cholesterol-reducing drugs is given by the slope of the straight line represented by M, which is approximately \$1.95 billion per year. ■

TECHNOLOGY EXERCISES

Find the value of y corresponding to the given value of x.

1. $3.1x + 2.4y - 12 = 0$; $x = 2.1$
2. $1.2x - 3.2y + 8.2 = 0$; $x = 1.2$
3. $2.8x + 4.2y = 16.3$; $x = 1.5$
4. $-1.8x + 3.2y - 6.3 = 0$; $x = -2.1$
5. $22.1x + 18.2y - 400 = 0$; $x = 12.1$
6. $17.1x - 24.31y - 512 = 0$; $x = -8.2$
7. $2.8x = 1.41y - 2.64$; $x = 0.3$
8. $0.8x = 3.2y - 4.3$; $x = -0.4$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 40

1.4 Intersection of Straight Lines

Finding the Point of Intersection

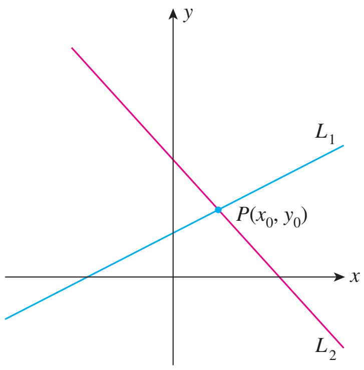


FIGURE 35
L_1 and L_2 intersect at the point $P(x_0, y_0)$.

The solution of certain practical problems involves finding the point of intersection of two straight lines. To see how such a problem may be solved algebraically, suppose we are given two straight lines L_1 and L_2 with equations

$$y = m_1x + b_1 \quad \text{and} \quad y = m_2x + b_2$$

(where m_1, b_1, m_2, and b_2 are constants) that intersect at the point $P(x_0, y_0)$ (Figure 35).

The point $P(x_0, y_0)$ lies on the line L_1, so it satisfies the equation $y = m_1x + b_1$. It also lies on the line L_2, so it satisfies the equation $y = m_2x + b_2$. Therefore, to find the point of intersection $P(x_0, y_0)$ of the lines L_1 and L_2, we solve the system composed of the two equations

$$y = m_1x + b_1 \quad \text{and} \quad y = m_2x + b_2$$

for x and y.

VIDEO

EXAMPLE 1 Find the point of intersection of the straight lines that have equations $y = x + 1$ and $y = -2x + 4$.

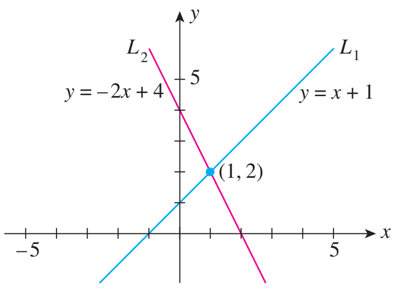


FIGURE 36
The point of intersection of L_1 and L_2 is (1, 2).

Solution We solve the given simultaneous equations. Substituting the value y as given in the first equation into the second, we obtain

$$\begin{aligned} x + 1 &= -2x + 4 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

Substituting this value of x into either one of the given equations yields $y = 2$. Therefore, the required point of intersection is (1, 2) (Figure 36). ■

Exploring with TECHNOLOGY

1. Use a graphing utility to plot the straight lines L_1 and L_2 with equations $y = 3x - 2$ and $y = -2x + 3$, respectively, on the same set of axes in the standard viewing window. Then use TRACE and ZOOM to find the point of intersection of L_1 and L_2. Repeat using the "intersection" function of your graphing utility.
2. Find the point of intersection of L_1 and L_2 algebraically.
3. Comment on the effectiveness of each method.

We now turn to some applications involving the intersections of pairs of straight lines.

Break-Even Analysis

Consider a firm with (linear) cost function $C(x)$, revenue function $R(x)$, and profit function $P(x)$ given by

$$\begin{aligned} C(x) &= cx + F \\ R(x) &= sx \\ P(x) &= R(x) - C(x) = (s - c)x - F \end{aligned}$$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 41

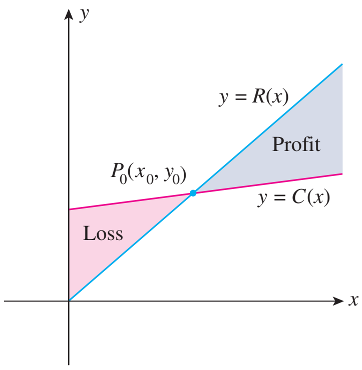


FIGURE 37
P_0 is the break-even point.

where c denotes the unit cost of production, s the selling price per unit, F the fixed cost incurred by the firm, and x the level of production and sales. The level of production at which the firm neither makes a profit nor sustains a loss is called the **break-even level of operation** and may be determined by solving the equations $y = C(x)$ and $y = R(x)$ simultaneously. At the level of production x_0, the profit is zero, so

$$P(x_0) = R(x_0) - C(x_0) = 0$$
$$R(x_0) = C(x_0)$$

The point $P_0(x_0, y_0)$, the solution of the simultaneous equations $y = R(x)$ and $y = C(x)$, is referred to as the **break-even point;** the number x_0 and the number y_0 are called the **break-even quantity** and the **break-even revenue,** respectively.

Geometrically, the break-even point $P_0(x_0, y_0)$ is just the point of intersection of the straight lines representing the cost and revenue functions, respectively. This follows because $P_0(x_0, y_0)$, being the solution of the simultaneous equations $y = R(x)$ and $y = C(x)$, must lie on both these lines simultaneously (Figure 37).

Note that if $x < x_0$, then $R(x) < C(x)$, so $P(x) = R(x) - C(x) < 0$; thus, the firm sustains a loss at this level of production. On the other hand, if $x > x_0$, then $P(x) > 0$, and the firm operates at a profitable level.

APPLIED EXAMPLE 2 Break-Even Level Prescott manufactures its products at a cost of \$4 per unit and sells them for \$10 per unit. If the firm's fixed cost is \$12,000 per month, determine the firm's break-even point.

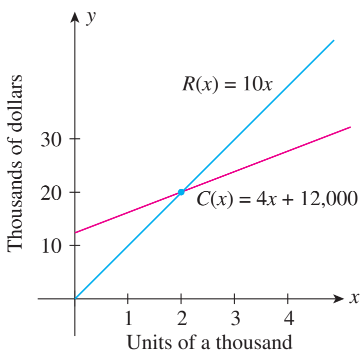


FIGURE 38
The point at which $R(x) = C(x)$ is the break-even point.

Solution The cost function C and the revenue function R are given by $C(x) = 4x + 12{,}000$ and $R(x) = 10x$, respectively (Figure 38).

Setting $R(x) = C(x)$, we obtain

$$10x = 4x + 12{,}000$$
$$6x = 12{,}000$$
$$x = 2000$$

Substituting this value of x into $R(x) = 10x$ gives

$$R(2000) = (10)(2000) = 20{,}000$$

So for a break-even operation, the firm should manufacture 2000 units of its product, resulting in a break-even revenue of \$20,000 per month. ■

APPLIED EXAMPLE 3 Break-Even Analysis Using the data given in Example 2, answer the following questions:

a. What is the loss sustained by the firm if only 1500 units are produced and sold each month?
b. What is the profit if 3000 units are produced and sold each month?
c. How many units should the firm produce to realize a minimum monthly profit of \$9000?

Solution The profit function P is given by the rule

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 10x - (4x + 12{,}000) \\ &= 6x - 12{,}000 \end{aligned}$$

a. If 1500 units are produced and sold each month, we have

$$P(1500) = 6(1500) - 12{,}000 = -3000$$

so the firm will sustain a loss of \$3000 per month.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 42

b. If 3000 units are produced and sold each month, we have

$$P(3000) = 6(3000) - 12{,}000 = 6000$$

or a monthly profit of \$6000.

c. Substituting 9000 for $P(x)$ in the equation $P(x) = 6x - 12{,}000$, we obtain

$$\begin{aligned} 9000 &= 6x - 12{,}000 \\ 6x &= 21{,}000 \\ x &= 3500 \end{aligned}$$

Thus, the firm should produce at least 3500 units to realize a \$9000 minimum monthly profit. ■

APPLIED EXAMPLE 4 Decision Analysis The management of Robertson Controls must decide between two manufacturing processes for its model C electronic thermostat. The monthly cost of the first process is given by $C_1(x) = 20x + 10{,}000$ dollars, where x is the number of thermostats produced; the monthly cost of the second process is given by $C_2(x) = 10x + 30{,}000$ dollars. If the projected monthly sales are 800 thermostats at a unit price of \$40, which process should management choose in order to maximize the company's profit?

Solution The break-even level of operation using the first process is obtained by solving the equation

$$\begin{aligned} 40x &= 20x + 10{,}000 \\ 20x &= 10{,}000 \\ x &= 500 \end{aligned}$$

giving an output of 500 units. Next, we solve the equation

$$\begin{aligned} 40x &= 10x + 30{,}000 \\ 30x &= 30{,}000 \\ x &= 1000 \end{aligned}$$

giving an output of 1000 units for a break-even operation using the second process. Since the projected sales are 800 units, we conclude that management should choose the first process, which will give the firm a profit. ■

APPLIED EXAMPLE 5 Decision Analysis Referring to Example 4, decide which process Robertson's management should choose if the projected monthly sales are (a) 1500 units and (b) 3000 units.

Solution In both cases, the production is past the break-even level. Since the revenue is the same regardless of which process is employed, the decision will be based on how much each process costs.

a. If $x = 1500$, then

$$\begin{aligned} C_1(x) &= (20)(1500) + 10{,}000 = 40{,}000 \\ C_2(x) &= (10)(1500) + 30{,}000 = 45{,}000 \end{aligned}$$

Hence, management should choose the first process.

b. If $x = 3000$, then

$$\begin{aligned} C_1(x) &= (20)(3000) + 10{,}000 = 70{,}000 \\ C_2(x) &= (10)(3000) + 30{,}000 = 60{,}000 \end{aligned}$$

In this case, management should choose the second process. ■

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 43

Exploring with TECHNOLOGY

1. Use a graphing utility to plot the straight lines L_1 and L_2 with equations $y = 2x - 1$ and $y = 2.1x + 3$, respectively, on the same set of axes, using the standard viewing window. Do the lines appear to intersect?
2. Plot the straight lines L_1 and L_2, using the viewing window $[-100, 100] \times [-100, 100]$. Do the lines appear to intersect? Can you find the point of intersection using TRACE and ZOOM? Using the "intersection" function of your graphing utility?
3. Find the point of intersection of L_1 and L_2 algebraically.
4. Comment on the effectiveness of the solution methods in parts 2 and 3.

Market Equilibrium

Under pure competition, the price of a commodity eventually settles at a level dictated by the condition that the supply of the commodity be equal to the demand for it. If the price is too high, consumers will be more reluctant to buy, and if the price is too low, the supplier will be more reluctant to make the product available in the marketplace. **Market equilibrium** is said to prevail when the quantity produced is equal to the quantity demanded. The quantity produced at market equilibrium is called the **equilibrium quantity**, and the corresponding price is called the **equilibrium price.**

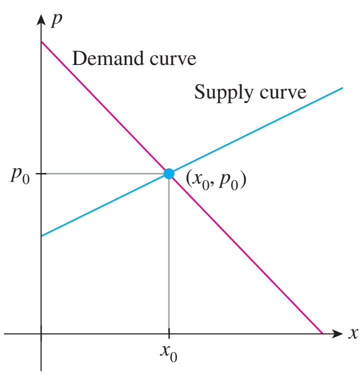


FIGURE 39
Market equilibrium is represented by the point (x_0, p_0).

From a geometric point of view, market equilibrium corresponds to the point at which the demand curve and the supply curve intersect. In Figure 39, x_0 represents the equilibrium quantity and p_0 the equilibrium price. The point (x_0, p_0) lies on the supply curve and therefore satisfies the supply equation. At the same time, it also lies on the demand curve and therefore satisfies the demand equation. Thus, to find the point (x_0, p_0), and hence the equilibrium quantity and price, we solve the demand and supply equations simultaneously for x and p. For meaningful solutions, x and p must both be positive.

APPLIED EXAMPLE 6 Market Equilibrium The management of ThermoMaster, which manufactures an indoor–outdoor thermometer at its Mexico subsidiary, has determined that the demand equation for its product is

$$5x + 3p - 30 = 0$$

where p is the price of a thermometer in dollars and x is the quantity demanded in units of a thousand. The supply equation for these thermometers is

$$52x - 30p + 45 = 0$$

where x (measured in thousands) is the quantity that ThermoMaster will make available in the market at p dollars each. Find the equilibrium quantity and price.

Solution We need to solve the system of equations

$$\begin{aligned} 5x + 3p - 30 &= 0 \\ 52x - 30p + 45 &= 0 \end{aligned}$$

for x and p. Let us use the *method of substitution* to solve it. As the name suggests, this method calls for choosing one of the equations in the system, solving

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 44

for one variable in terms of the other, and then substituting the resulting expression into the other equation. This gives an equation in one variable that can then be solved in the usual manner.

Let's solve the first equation for p in terms of x. Thus,

$$\begin{aligned} 3p &= -5x + 30 \\ p &= -\frac{5}{3}x + 10 \end{aligned}$$

Next, we substitute this value of p into the second equation, obtaining

$$\begin{aligned} 52x - 30\left(-\frac{5}{3}x + 10\right) + 45 &= 0 \\ 52x + 50x - 300 + 45 &= 0 \\ 102x - 255 &= 0 \\ x &= \frac{255}{102} = \frac{5}{2} \end{aligned}$$

The corresponding value of p is found by substituting this value of x into the equation for p obtained earlier. Thus,

$$\begin{aligned} p &= -\frac{5}{3}\left(\frac{5}{2}\right) + 10 = -\frac{25}{6} + 10 \\ &= \frac{35}{6} \approx 5.83 \end{aligned}$$

We conclude that the equilibrium quantity is 2500 units (remember that x is measured in units of a thousand) and the equilibrium price is \$5.83 per thermometer.

VIDEO

APPLIED EXAMPLE 7 Market Equilibrium The quantity demanded of a certain model of DVD player is 8000 units when the unit price is \$260. At a unit price of \$200, the quantity demanded increases to 10,000 units. The manufacturer will not market any players if the price is \$100 or lower. However, for each \$50 increase in the unit price above \$100, the manufacturer will market an additional 1000 units. Both the demand and the supply equations are known to be linear.

a. Find the demand equation.
b. Find the supply equation.
c. Find the equilibrium quantity and price.

Solution Let p denote the unit price in hundreds of dollars, and let x denote the number of units of players in thousands.

a. Since the demand function is linear, the demand curve is a straight line passing through the points (8, 2.6) and (10, 2). Its slope is

$$m = \frac{2 - 2.6}{10 - 8} = -0.3$$

Using the point (10, 2) and the slope $m = -0.3$ in the point-slope form of the equation of a line, we see that the required demand equation is

$$\begin{aligned} p - 2 &= -0.3(x - 10) \\ p &= -0.3x + 5 \end{aligned}$$

Figure 40

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 45

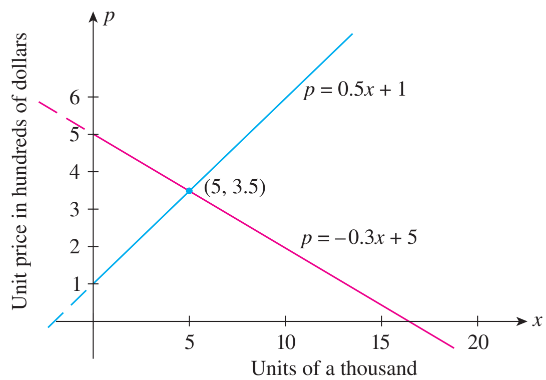


FIGURE 40
Market equilibrium occurs at the point (5, 3.5).

b. The supply curve is the straight line passing through the points (0, 1) and (1, 1.5). Its slope is

$$m = \frac{1.5 - 1}{1 - 0} = 0.5$$

Using the point (0, 1) and the slope $m = 0.5$ in the point-slope form of the equation of a line, we see that the required supply equation is

$$p - 1 = 0.5(x - 0)$$
$$p = 0.5x + 1 \qquad \text{Figure 40}$$

c. To find the market equilibrium, we solve simultaneously the system comprising the demand and supply equations obtained in parts (a) and (b)—that is, the system

$$\begin{aligned} p &= -0.3x + 5 \\ p &= 0.5x + 1 \end{aligned} \qquad \begin{aligned} p &= 0.5x + 1 \\ -p &= 0.3x - 5 \\ \hline 0 &= 0.8x - 4 \end{aligned}$$

Subtracting the first equation from the second gives

$$0.8x - 4 = 0$$

and $x = 5$. Substituting this value of x in the second equation gives $p = 3.5$. Thus, the equilibrium quantity is 5000 units, and the equilibrium price is \$350 (Figure 40). ■

1.4 Self-Check Exercises

1. Find the point of intersection of the straight lines with equations $2x + 3y = 6$ and $x - 3y = 4$.

2. There is no demand for a certain model of a disposable camera when the unit price is \$12. However, when the unit price is \$8, the quantity demanded is 8000/week. The suppliers will not market any cameras if the unit price is \$2 or lower. At \$4/camera, however, the manufacturer will market 5000 cameras/week. Both the demand and supply functions are known to be linear.
   a. Find the demand equation.
   b. Find the supply equation.
   c. Find the equilibrium quantity and price.

Solutions to Self-Check Exercises 1.4 can be found on page 48.

1.4 Concept Questions

1. Explain why you would expect that the intersection of a linear demand curve and a linear supply curve would lie in the first quadrant.

2. Explain the meaning of each term:
   a. Break-even point
   b. Break-even quantity
   c. Break-even revenue

3. Explain the meaning of each term:
   a. Market equilibrium
   b. Equilibrium quantity
   c. Equilibrium price

1.4 Exercises

In Exercises 1–6, find the point of intersection of each pair of straight lines.

1. $y = 3x + 4$
$y = -2x + 14$

2. $y = -4x - 7$
$-y = 5x + 10$

3. $2x - 3y = 6$
$3x + 6y = 16$

4. $2x + 4y = 11$
$-5x + 3y = 5$

5. $y = \frac{1}{4}x - 5$
$2x - \frac{3}{2}y = 1$

6. $y = \frac{2}{3}x - 4$
$x + 3y + 3 = 0$

In Exercises 7–10, find the break-even point for the firm whose cost function C and revenue function R are given.

7. $C(x) = 5x + 10{,}000$; $R(x) = 15x$

8. $C(x) = 15x + 12{,}000$; $R(x) = 21x$

9. $C(x) = 0.2x + 120$; $R(x) = 0.4x$

10. $C(x) = 150x + 20{,}000$; $R(x) = 270x$

11. Break-Even Analysis AutoTime, a manufacturer of 24-hr variable timers, has a monthly fixed cost of \$48,000 and a production cost of \$8 for each timer manufactured. The units sell for \$14 each.
a. Sketch the graphs of the cost function and the revenue function and thereby find the break-even point graphically.
b. Find the break-even point algebraically.
c. Sketch the graph of the profit function.
d. At what point does the graph of the profit function cross the x-axis? Interpret your result.

12. Break-Even Analysis A division of Carter Enterprises produces "Personal Income Tax" diaries. Each diary sells for \$8. The monthly fixed costs incurred by the division are \$25,000, and the variable cost of producing each diary is \$3.
a. Find the break-even point for the division.
b. What should be the level of sales in order for the division to realize a 15% profit over the cost of making the diaries?

13. Break-Even Analysis A division of the Gibson Corporation manufactures bicycle pumps. Each pump sells for \$9, and the variable cost of producing each unit is 40% of the selling price. The monthly fixed costs incurred by the division are \$50,000. What is the break-even point for the division?

14. Leasing Ace Truck Leasing Company leases a certain size truck for \$30/day and \$.15/mi, whereas Acme Truck Leasing Company leases the same size truck for \$25/day and \$.20/mi.
a. Find the functions describing the daily cost of leasing from each company.
b. Sketch the graphs of the two functions on the same set of axes.
c. If a customer plans to drive at most 70 mi, from which company should he rent a truck for a single day?

15. Decision Analysis A product may be made by using Machine I or Machine II. The manufacturer estimates that the monthly fixed costs of using Machine I are \$18,000, whereas the monthly fixed costs of using Machine II are \$15,000. The variable costs of manufacturing 1 unit of the product using Machine I and Machine II are \$15 and \$20, respectively. The product sells for \$50 each.
a. Find the cost functions associated with using each machine.
b. Sketch the graphs of the cost functions of part (a) and the revenue functions on the same set of axes.
c. Which machine should management choose in order to maximize their profit if the projected sales are 450 units? 550 units? 650 units?
d. What is the profit for each case in part (c)?

16. Annual Sales The annual sales of Crimson Drug Store are expected to be given by $S = 2.3 + 0.4t$ million dollars t years from now, whereas the annual sales of Cambridge Drug Store are expected to be given by $S = 1.2 + 0.6t$ million dollars t years from now. When will Cambridge's annual sales first surpass Crimson's annual sales?

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 47

17. LCDs Versus CRTs The global shipments of traditional cathode-ray tube monitors (CRTs) is approximated by the equation

$$y = -12t + 88 \qquad (0 \le t \le 3)$$

where y is measured in millions and t in years, with $t = 0$ corresponding to the beginning of 2001. The equation

$$y = 18t + 13.4 \qquad (0 \le t \le 3)$$

gives the approximate number (in millions) of liquid crystal displays (LCDs) over the same period. When did the global shipments of LCDs first overtake the global shipments of CRTs?

Source: International Data Corporation.

18. Digital Versus Film Cameras The sales of digital cameras (in millions of units) in year t is given by the function

$$f(t) = 3.05t + 6.85 \qquad (0 \le t \le 3)$$

where $t = 0$ corresponds to 2001. Over that same period, the sales of film cameras (in millions of units) is given by

$$g(t) = -1.85t + 16.58 \qquad (0 \le t \le 3)$$

a. Show that more film cameras than digital cameras were sold in 2001.
b. When did the sales of digital cameras first exceed those of film cameras?

Source: Popular Science.

19. U.S. Financial Transactions The percentage of U.S. transactions by check between the beginning of 2001 ($t = 0$) and the beginning of 2010 ($t = 9$) is approximated by

$$f(t) = -\frac{11}{9}t + 43 \qquad (0 \le t \le 9)$$

whereas the percentage of transactions done electronically during the same period is approximated by

$$g(t) = \frac{11}{3}t + 23 \qquad (0 \le t \le 9)$$

a. Sketch the graphs of f and g on the same set of axes.
b. Find the time when transactions done electronically first exceeded those done by check.

Source: Foreign Policy.

20. Broadband Versus Dial-Up The number of U.S. broadband Internet households (in millions) between the beginning of 2004 ($t = 0$) and the beginning of 2008 ($t = 4$) is approximated by

$$f(t) = 6.5t + 33 \qquad (0 \le t \le 4)$$

Over the same period, the number of U.S. dial-up Internet households (in millions) is approximated by

$$g(t) = -3.9t + 42.5 \qquad (0 \le t \le 4)$$

a. Sketch the graphs of f and g on the same set of axes.
b. Solve the equation $f(t) = g(t)$, and interpret your result.

Source: Strategic Analytics, Inc.

For each pair of supply-and-demand equations in Exercises 21–24, where x represents the quantity demanded in units of 1000 and p is the unit price in dollars, find the equilibrium quantity and the equilibrium price.

21. $4x + 3p - 59 = 0$ and $5x - 6p + 14 = 0$

22. $2x + 7p - 56 = 0$ and $3x - 11p + 45 = 0$

23. $p = -2x + 22$ and $p = 3x + 12$

24. $p = -0.3x + 6$ and $p = 0.15x + 1.5$

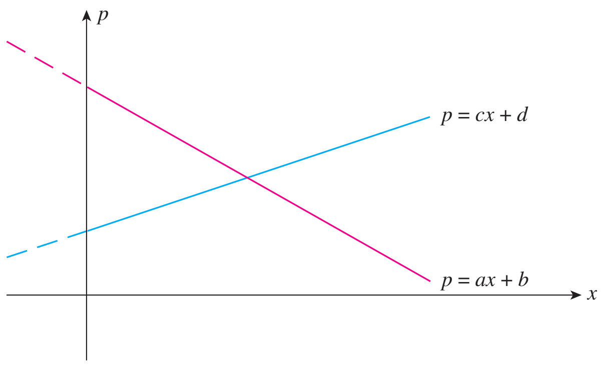
25. Equilibrium Quantity and Price The quantity demanded of a certain brand of DVD player is 3000/week when the unit price is \$485. For each decrease in unit price of \$20 below \$485, the quantity demanded increases by 250 units. The suppliers will not market any DVD players if the unit price is \$300 or lower. But at a unit price of \$525, they are willing to make available 2500 units in the market. The supply equation is also known to be linear.
a. Find the demand equation.
b. Find the supply equation.
c. Find the equilibrium quantity and price.

26. Equilibrium Quantity and Price The demand equation for the Drake GPS Navigator is $x + 4p - 800 = 0$, where x is the quantity demanded per week and p is the wholesale unit price in dollars. The supply equation is $x - 20p + 1000 = 0$, where x is the quantity the supplier will make available in the market each week when the wholesale price is p dollars each. Find the equilibrium quantity and the equilibrium price for the GPS Navigators.

27. Equilibrium Quantity and Price The demand equation for the Schmidt-3000 fax machine is $3x + p - 1500 = 0$, where x is the quantity demanded per week and p is the unit price in dollars. The supply equation is $2x - 3p + 1200 = 0$, where x is the quantity the supplier will make available in the market each week when the unit price is p dollars. Find the equilibrium quantity and the equilibrium price for the fax machines.

28. Equilibrium Quantity and Price The quantity demanded each month of Russo Espresso Makers is 250 when the unit price is \$140; the quantity demanded each month is 1000 when the unit price is \$110. The suppliers will market 750 espresso makers if the unit price is \$60 or higher. At a unit price of \$80, they are willing to market 2250 units. Both the demand and supply equations are known to be linear.
a. Find the demand equation.
b. Find the supply equation.
c. Find the equilibrium quantity and the equilibrium price.

29. Suppose the demand and supply equations for a certain commodity are given by $p = ax + b$ and $p = cx + d$, respectively, where $a < 0$, $c > 0$, and $b > d > 0$ (see the following figure).



a. Find the equilibrium quantity and equilibrium price in terms of a, b, c, and d.
b. Use part (a) to determine what happens to the market equilibrium if c is increased while a, b, and d remain fixed. Interpret your answer in economic terms.
c. Use part (a) to determine what happens to the market equilibrium if b is decreased while a, c, and d remain fixed. Interpret your answer in economic terms.

30. Suppose the cost function associated with a product is $C(x) = cx + F$ dollars and the revenue function is $R(x) = sx$, where c denotes the unit cost of production, s the unit selling price, F the fixed cost incurred by the firm, and x the level of production and sales. Find the break-even quantity and the break-even revenue in terms of the constants c, s, and F, and interpret your results in economic terms.

In Exercises 31 and 32, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

31. Suppose $C(x) = cx + F$ and $R(x) = sx$ are the cost and revenue functions, respectively, of a certain firm. Then the firm is operating at a break-even level of production if its level of production is $F/(s - c)$.

32. If both the demand equation and the supply equation for a certain commodity are linear, then there must be at least one equilibrium point.

33. Let L_1 and L_2 be two nonvertical straight lines in the plane with equations $y = m_1x + b_1$ and $y = m_2x + b_2$, respectively. Find conditions on m_1, m_2, b_1, and b_2 such that (a) L_1 and L_2 do not intersect, (b) L_1 and L_2 intersect at one and only one point, and (c) L_1 and L_2 intersect at infinitely many points.

34. Find conditions on a_1, a_2, b_1, b_2, c_1, and c_2 such that the system of linear equations

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

has (a) no solution, (b) a unique solution, and (c) infinitely many solutions.
Hint: Use the results of Exercise 33.

1.4 Solutions to Self-Check Exercises

1. The point of intersection of the two straight lines is found by solving the system of linear equations

$$2x + 3y = 6$$
$$x - 3y = 4$$

Solving the first equation for y in terms of x, we obtain

$$y = -\frac{2}{3}x + 2$$

Substituting this expression for y into the second equation, we obtain

$$x - 3\left(-\frac{2}{3}x + 2\right) = 4$$
$$x + 2x - 6 = 4$$
$$3x = 10$$

or $x = \frac{10}{3}$. Substituting this value of x into the expression for y obtained earlier, we find

$$y = -\frac{2}{3}\left(\frac{10}{3}\right) + 2 = -\frac{2}{9}$$

Therefore, the point of intersection is $(\frac{10}{3}, -\frac{2}{9})$.

2. a. Let p denote the price per camera, and let x denote the quantity demanded per week. The given conditions imply that $x = 0$ when $p = 12$ and $x = 8000$ when $p = 8$. Since the demand equation is linear, it has the form

$$p = mx + b$$

Now, the first condition implies that

$$12 = m(0) + b \quad \text{or} \quad b = 12$$

Therefore,

$$p = mx + 12$$

Using the second condition, we find

$$8 = 8000m + 12$$
$$m = -\frac{4}{8000} = -0.0005$$

Hence, the required demand equation is

$$p = -0.0005x + 12$$

b. Let p denote the price per camera, and let x denote the quantity made available at that price per week. Then, since the supply equation is linear, it also has the form

$$p = mx + b$$

48149_01_ch1_p001-066 11/3/10 11:36 AM Page 49

The first condition implies that $x = 0$ when $p = 2$, so we have

$$2 = m(0) + b \quad \text{or} \quad b = 2$$

Therefore,

$$p = mx + 2$$

Next, using the second condition, $x = 5000$ when $p = 4$, we find

$$4 = 5000m + 2$$

giving $m = 0.0004$. So the required supply equation is

$$p = 0.0004x + 2$$

c. The equilibrium quantity and price are found by solving the system of linear equations

$$p = -0.0005x + 12$$
$$p = 0.0004x + 2$$

Equating the two expressions yields

$$-0.0005x + 12 = 0.0004x + 2$$
$$0.0009x = 10$$

or $x \approx 11{,}111$. Substituting this value of x into either equation in the system yields

$$p \approx 6.44$$

Therefore, the equilibrium quantity is 11,111, and the equilibrium price is $6.44.

USING TECHNOLOGY

Finding the Point(s) of Intersection of Two Graphs

Graphing Utility

A graphing utility can be used to find the point(s) of intersection of the graphs of two functions. Once again, it is important to remember that if the graphs are straight lines, the linear equations defining these lines must first be recast in the slope-intercept form.

EXAMPLE 1 Find the points of intersection of the straight lines with equations $2x + 3y = 6$ and $3x - 4y - 5 = 0$.

Solution **Method I** Solving each equation for y in terms of x, we obtain

$$y = -\frac{2}{3}x + 2 \quad \text{and} \quad y = \frac{3}{4}x - \frac{5}{4}$$

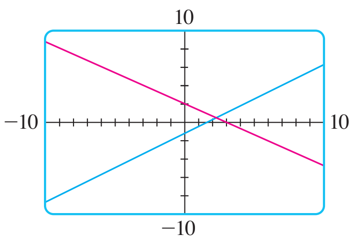


FIGURE T1
The straight lines $2x + 3y = 6$ and $3x - 4y - 5 = 0$

as the respective equations in the slope-intercept form. The graphs of the two straight lines in the standard viewing window are shown in Figure T1.

Then, using TRACE and ZOOM or the function for finding the point of intersection of two graphs, we find that the point of intersection, accurate to four decimal places, is (2.2941, 0.4706).

Method II Proceed as before to obtain the graphs of the two lines. Then use the intersect function of the graphing utility to find the point of intersection, (2.2941176, 0.47058824), of the two straight lines (Figure T2).

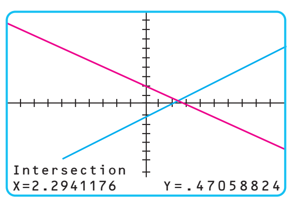


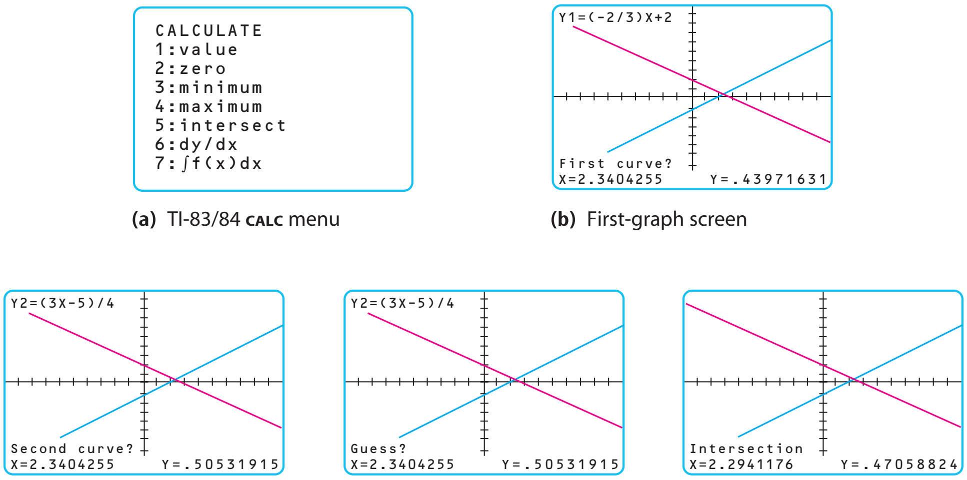
FIGURE T2

Note On the TI-83/84, you can call the **intersect** function by selecting the CALC menu and then selecting **5: intersect** (Figure T3a). Press ENTER to obtain the graph

(continued)

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 50

shown in Figure T3b. Then press **ENTER** to select the first curve (Figure T3c); press **ENTER** again to select the second curve (Figure T3d). Press **ENTER** for the fourth time to find the point of intersection (Figure T3e).



(a) TI-83/84 CALC menu

(b) First-graph screen

(c) Second-graph screen

(d) Third-graph screen

(e) Final-graph screen

FIGURE **T3**

TECHNOLOGY EXERCISES

In Exercises 1–6, find the point of intersection of the pair of straight lines with the given equations. Round your answers to four decimal places.

1. $y = 2x + 5$ and $y = -3x + 8$
2. $y = 1.2x + 6.2$ and $y = -4.3x + 9.1$
3. $2x - 5y = 7$ and $3x + 4y = 12$
4. $1.4x - 6.2y = 8.4$ and $4.1x + 7.3y = 14.4$
5. $2.1x = 5.1y + 71$ and $3.2x = 8.4y + 16.8$
6. $8.3x = 6.2y + 9.3$ and $12.4x = 12.3y + 24.6$
7. **Break-Even Analysis** PhotoMax makes disposable cameras that sell for \$7.89 each and cost \$3.24 each to produce. The weekly fixed cost for the company is \$16,500.
   a. Plot the graphs of the cost function and the revenue function in the viewing window $[0, 6000] \times [0, 60{,}000]$.
   b. Find the break-even point by using the viewing window $[0, 6000] \times [-20{,}000, 20{,}000]$.
   c. Plot the graph of the profit function and verify the result of part (b) by finding the x-intercept.
8. **Break-Even Analysis** The Monde Company makes a wine cooler with a capacity of 24 bottles. Each wine cooler sells for \$245. The monthly fixed costs incurred by the company are \$385,000, and the variable cost of producing each wine cooler is \$90.50.
   a. Find the break-even point for the company.
   b. Find the level of sales needed to ensure that the company will realize a profit of 21% over the cost of producing the wine coolers.
9. **Leasing** Ace Truck Leasing Company leases a certain size truck for \$34/day and \$0.18/mi, whereas Acme Truck Leasing Company leases the same size truck for \$28/day and \$0.22/mi.
   a. Find the functions describing the daily cost of leasing from each company.
   b. Plot the graphs of the two functions using the same viewing window.
   c. Find the point of intersection of the graphs of part (b).
   d. Use the result of part (c) to find a criterion that a customer can use to help her decide which company to rent the truck from if she knows the maximum distance that she will drive on the day of rental.
10. **Bank Deposits** The total deposits with a branch of Randolph Bank currently stand at \$20.384 million and are projected to grow at the rate of \$1.019 million/year for the next 5 years. The total deposits with a branch of Madison Bank, in the same shopping complex as the Randolph Bank, currently stand at \$18.521 million and are expected to grow at the rate of \$1.482 million/year for the next 5 years.
   a. Find the function describing the total deposits with each bank for the next 5 years.
   b. Plot the graphs of the two functions found in part (a) using the same viewing window.
   c. Do the total deposits of Madison catch up to those of Randolph over the period in question? If so, at what time?

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 51

11. Equilibrium Quantity and Price The quantity demanded of a certain brand of a smart phone is 2000/week when the unit price is \$84. For each decrease in unit price of \$5 below \$84, the quantity demanded increases by 50 units. The supplier will not market any of the smart phones if the unit price is \$60 or less, but the supplier will market 1800/week if the unit price is \$90. The supply equation is also known to be linear.

a. Find the demand and supply equations.

b. Plot the graphs of the supply and demand equations and find their point of intersection.

c. Find the equilibrium quantity and price.

12. Equilibrium Quantity and Price The demand equation for the Miramar Heat Machine, a ceramic heater, is $1.1x + 3.8p - 901 = 0$, where x is the quantity demanded each week and p is the wholesale unit price in dollars. The corresponding supply equation is $0.9x - 20.4p + 1038 = 0$, where x is the quantity the supplier will make available in the market when the wholesale price is p dollars each.

a. Plot the graphs of the demand and supply equations using the same viewing window.

b. Find the equilibrium quantity and the equilibrium price for the Miramar heaters.

1.5 The Method of Least Squares

The Method of Least Squares

In Example 10, Section 1.2, we saw how a linear equation may be used to approximate the sales trend for a local sporting goods store. The *trend line*, as we saw, may be used to predict the store's future sales. Recall that we obtained the trend line in Example 10 by requiring that the line pass through two data points, the rationale being that such a line seems to *fit* the data reasonably well.

In this section, we describe a general method known as the **method of least squares** for determining a straight line that, in some sense, best fits a set of data points when the points are scattered about a straight line. To illustrate the principle behind the method of least squares, suppose, for simplicity, that we are given five data points,

$$P_1(x_1, y_1),\ P_2(x_2, y_2),\ P_3(x_3, y_3),\ P_4(x_4, y_4),\ P_5(x_5, y_5)$$

describing the relationship between the two variables x and y. By plotting these data points, we obtain a graph called a **scatter diagram** (Figure 41).

If we try to *fit* a straight line to these data points, the line will miss the first, second, third, fourth, and fifth data points by the amounts d_1, d_2, d_3, d_4, and d_5, respectively (Figure 42). We can think of the amounts $d_1, d_2, \ldots, d_5$ as the errors made when the values $y_1, y_2, \ldots, y_5$ are approximated by the corresponding values of y lying on the straight line L.

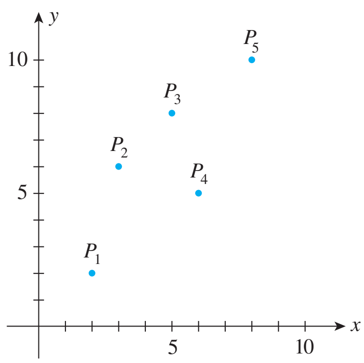


FIGURE 41
A scatter diagram

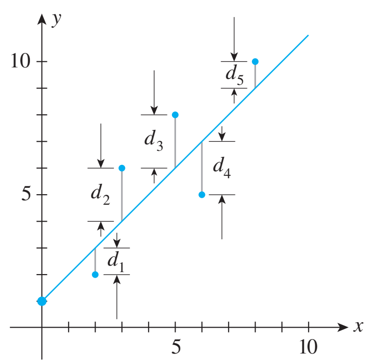


FIGURE 42
d_i is the vertical distance between the straight line and a given data point.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 52

PORTFOLIO

Melissa Rich

TITLE Owner
INSTITUTION A Rich Experience Massage & Wellness Spa

A Rich Experience Massage & Wellness Spa takes an integrative approach to health-related services in a modern spa environment. The spa provides a variety of services including massage therapy, spa facials and body treatments, Bowenwork, hypnotherapy, and chiropractic care. A Rich Experience also sells retail products and offers continuing education workshops for massage therapists.

As an owner, I closely evaluate the company's fixed and variable revenue streams—generated from our beauty and spa services, retail sales, workshops, and rental space—to determine if I am on track to meet my profitability goals. I plot data points for all the revenue streams and apply the least squares method to see if the best fit line matches my profitability goals. This method allows me to see how increases in specific revenue streams might affect overall profitability.

Bodywork and Spa Services provide us with our most profitable revenue stream, so I try to maximize the number of these services that we can provide. Based on our available rooms and practitioners, we can provide 60 one-hour services per week. At an average of $70 per service, this equals $4200 per week or approximately $218,440 per year. In order to gross $109,220 and meet my goals for net profitability, after one year the spa needs to be booking at least 50% of our available time slots. My commissioned practitioners earn 40% on all services they provide, so at 50% productivity, this brings my net total for Beauty and Spa Services to $65,532.

Applying my college mathematics and accounting courses has helped me create a thriving business. With additional marketing efforts, name recognition, and a positive reputation in the community, I can anticipate that our bookings will continue to increase each year.

Courtesy of A Rich Experience; (inset) © Szasz-Fabian Ilka Erica/Shutterstock.com

The **principle of least squares** states that the straight line L that fits the data points *best* is the one chosen by requiring that the sum of the squares of $d_1, d_2, \ldots, d_5$—that is,

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

be made as small as possible. In other words, the least-squares criterion calls for minimizing the sum of the squares of the errors. The line L obtained in this manner is called the **least-squares line,** or *regression line.*

The method for computing the least-squares lines that best fits a set of data points is contained in the following result, which we state without proof.

The Method of Least Squares

Suppose we are given n data points

$$P_1(x_1, y_1),\ P_2(x_2, y_2),\ P_3(x_3, y_3), \ldots, P_n(x_n, y_n)$$

Then the least-squares (regression) line for the data is given by the linear equation (function)

$$y = f(x) = mx + b$$

where the constants m and b satisfy the **normal equations**

$$nb + (x_1 + x_2 + \cdots + x_n)m = y_1 + y_2 + \cdots + y_n \qquad \textbf{(9)}$$

$$(x_1 + x_2 + \cdots + x_n)b + (x_1^2 + x_2^2 + \cdots + x_n^2)m = x_1y_1 + x_2y_2 + \cdots + x_ny_n \qquad \textbf{(10)}$$

simultaneously.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 53

VIDEO **EXAMPLE 1** Find the least-squares line for the data

$$P_1(1, 1), P_2(2, 3), P_3(3, 4), P_4(4, 3), P_5(5, 6)$$

Solution Here, we have $n = 5$ and

$$x_1 = 1 \quad x_2 = 2 \quad x_3 = 3 \quad x_4 = 4 \quad x_5 = 5$$
$$y_1 = 1 \quad y_2 = 3 \quad y_3 = 4 \quad y_4 = 3 \quad y_5 = 6$$

Before using Equations (9) and (10), it is convenient to summarize these data in the form of a table:

	x	y	x^2	xy
	1	1	1	1
	2	3	4	6
	3	4	9	12
	4	3	16	12
	5	6	25	30
Sum	15	17	55	61

Using this table and (9) and (10), we obtain the normal equations

$$5b + 15m = 17 \qquad \textbf{(11)}$$
$$15b + 55m = 61 \qquad \textbf{(12)}$$

Solving Equation (11) for b gives

$$b = -3m + \frac{17}{5} \qquad \textbf{(13)}$$

which, upon substitution into Equation (12), gives

$$\begin{aligned} 15\left(-3m + \frac{17}{5}\right) + 55m &= 61 \\ -45m + 51 + 55m &= 61 \\ 10m &= 10 \\ m &= 1 \end{aligned}$$

Substituting this value of m into Equation (13) gives

$$b = -3 + \frac{17}{5} = \frac{2}{5} = 0.4$$

Therefore, the required least-squares line is

$$y = x + 0.4$$

The scatter diagram and the least-squares line are shown in Figure 43. ■

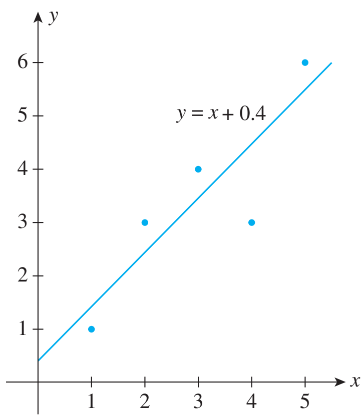


FIGURE **43**
The least-squares line $y = x + 0.4$ and the given data points

VIDEO  **APPLIED EXAMPLE 2** Advertising and Profit The proprietor of Leisure Travel Service compiled the following data relating the annual profit of the firm to its annual advertising expenditure (both measured in thousands of dollars):

Annual Advertising Expenditure, x	12	14	17	21	26	30
Annual Profit, y	60	70	90	100	100	120

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 54

a. Determine the equation of the least-squares line for these data.
b. Draw a scatter diagram and the least-squares line for these data.
c. Use the result obtained in part (a) to predict Leisure Travel's annual profit if the annual advertising budget is \$20,000.

Solution

a. The calculations required for obtaining the normal equations are summarized in the following table:

	x	y	x^2	xy
	12	60	144	720
	14	70	196	980
	17	90	289	1,530
	21	100	441	2,100
	26	100	676	2,600
	30	120	900	3,600
Sum	120	540	2646	11,530

The normal equations are

$$6b + 120m = 540 \qquad \textbf{(14)}$$
$$120b + 2646m = 11{,}530 \qquad \textbf{(15)}$$

Solving Equation (14) for b gives

$$b = -20m + 90 \qquad \textbf{(16)}$$

which, upon substitution into Equation (15), gives

$$\begin{aligned} 120(-20m + 90) + 2646m &= 11{,}530 \\ -2400m + 10{,}800 + 2646m &= 11{,}530 \\ 246m &= 730 \\ m &\approx 2.97 \end{aligned}$$

Substituting this value of m into Equation (16) gives

$$b = -20(2.97) + 90 = 30.6$$

Therefore, the required least-squares line is given by

$$y = f(x) = 2.97x + 30.6$$

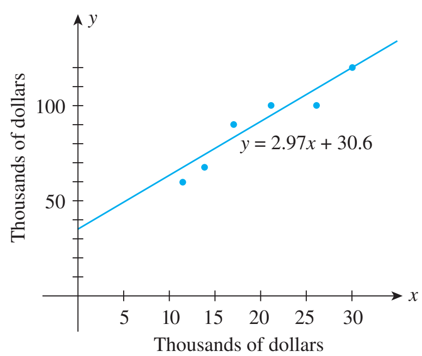


FIGURE 44
Profit versus advertising expenditure

b. The scatter diagram and the least-squares line are shown in Figure 44.
c. Leisure Travel's predicted annual profit corresponding to an annual budget of \$20,000 is given by

$$f(20) = 2.97(20) + 30.6 = 90$$

or \$90,000. ■

APPLIED EXAMPLE 3 U.S. Health-Care Expenditures Refer to Example 1 of Section 1.3. Because the over-65 population will be growing more rapidly in the next few decades, health-care spending is expected to increase significantly in the coming decades. The following table gives the U.S. health expenditures (in trillions of dollars) from 2008 through 2013, where t is measured in years, with $t = 0$ corresponding to 2008.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 55

Year, t	0	1	2	3	4	5
Expenditure, y	2.34	2.47	2.57	2.70	2.85	3.02

(The figures after 2009 are estimates.) Find a function giving the U.S. health-care spending between 2008 and 2013, using the least-squares technique.
Source: Centers for Medicare & Medicaid Services.

Solution The calculations required for obtaining the normal equations are summarized in the following table:

t	y	t^2	ty
0	2.34	0	0
1	2.47	1	2.47
2	2.57	4	5.14
3	2.70	9	8.10
4	2.85	16	11.40
5	3.02	25	15.10
15	15.95	55	42.21

The normal equations are

$$6b + 15m = 15.95 \quad (17)$$

$$15b + 55m = 42.21 \quad (18)$$

Solving Equation (17) for b gives

$$b \approx -2.5m + 2.6583 \quad (19)$$

which, upon substitution into Equation (18), gives

$$\begin{aligned} 15(-2.5m + 2.6583) + 55m &= 42.21 \\ -37.5m + 39.8745 + 55m &= 42.21 \\ 17.5m &= 2.3355 \\ m &\approx 0.1335 \end{aligned}$$

Substituting this value of m into Equation (19) gives

$$b \approx -2.5(0.1335) + 2.6583 \approx 2.3246$$

Therefore, the required function is

$$S(t) = 0.134t + 2.325$$

■

1.5 Self-Check Exercises

1. Find an equation of the least-squares line for the data

x	1	3	4	5	7
y	4	10	11	12	16

2. In a market research study for Century Communications, the following data were provided based on the projected monthly sales x (in thousands) of a DVD version of a box-office-hit adventure movie with a proposed wholesale unit price of p dollars.

x	2.2	5.4	7.0	11.5	14.6
p	38.0	36.0	34.5	30.0	28.5

Find the demand equation if the demand curve is the least-squares line for these data.

Solutions to Self-Check Exercises 1.5 can be found on page 59.

1.5 Concept Questions

1. Explain the terms (a) *scatter diagram* and (b) *least-squares line.*

2. State the principle of least squares in your own words.

1.5 Exercises

In Exercises 1–6, (a) find the equation of the least-squares line for the data, and (b) draw a scatter diagram for the data and graph the least-squares line.

1.

x	1	2	3	4
y	4	6	8	11

2.

x	1	3	5	7	9
y	9	8	6	3	2

3.

x	1	2	3	4	4	6
y	4.5	5	3	2	3.5	1

4.

x	1	1	2	3	4	4	5
y	2	3	3	3.5	3.5	4	5

5. $P_1(1, 3)$, $P_2(2, 5)$, $P_3(3, 5)$, $P_4(4, 7)$, $P_5(5, 8)$

6. $P_1(1, 8)$, $P_2(2, 6)$, $P_3(5, 6)$, $P_4(7, 4)$, $P_5(10, 1)$

7. **College Admissions** The accompanying data were compiled by the admissions office at Faber College during the past 5 years. The data relate the number of college brochures and follow-up letters (x) sent to a preselected list of high school juniors who had taken the PSAT and the number of completed applications (y) received from these students (both measured in units of a thousand).

x	4	4.5	5	5.5	6
y	0.5	0.6	0.8	0.9	1.2

a. Determine the equation of the least-squares line for these data.
b. Draw a scatter diagram and the least-squares line for these data.
c. Use the result obtained in part (a) to predict the number of completed applications expected if 6400 brochures and follow-up letters are sent out during the next year.

8. **Bounced-Check Charges** Overdraft fees have become an important piece of a bank's total fee income. The following table gives the bank revenue from overdraft fees (in billions of dollars) from 2004 through 2009. Here, $x = 4$ corresponds to 2004.

Year, x	4	5	6	7	8	9
Revenue, y	27.5	29	31	34	36	38

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the average rate of increase in overdraft fees over the period under consideration.
c. Assuming that the trend continued, what was the revenue from overdraft fees in 2011?

Source: New York Times.

9. **SAT Verbal Scores** The accompanying data were compiled by the superintendent of schools in a large metropolitan area. The table shows the average SAT verbal scores of high school seniors during the 5 years since the district implemented its "back to basics" program.

Year, x	1	2	3	4	5
Average Score, y	436	438	428	430	426

a. Determine the equation of the least-squares line for these data.
b. Draw a scatter diagram and the least-squares line for these data.
c. Use the result obtained in part (a) to predict the average SAT verbal score of high school seniors 2 years from now ($x = 7$).

10. **Net Sales** The management of Kaldor, a manufacturer of electric motors, submitted the accompanying data in its annual stockholders report. The following table shows the net sales (in millions of dollars) during the 5 years that have elapsed since the new management team took over:

Year, x	1	2	3	4	5
Net Sales, y	426	437	460	473	477

(The first year the firm operated under the new management corresponds to the time period $x = 1$, and the four subsequent years correspond to $x = 2, 3, 4$, and 5.)
a. Determine the equation of the least-squares line for these data.
b. Draw a scatter diagram and the least-squares line for these data.
c. Use the result obtained in part (a) to predict the net sales for the upcoming year.

11. **Mass-Transit Subsidies** The following table gives the projected state subsidies (in millions of dollars) to the Massachusetts Bay Transit Authority (MBTA) over a 5-year period:

48149_01_ch1_p001-066 11/16/10 9:26 AM Page 57

Year, x	1	2	3	4	5
Subsidy, y	20	24	26	28	32

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the state subsidy to the MBTA for the eighth year ($x = 8$).

Source: Massachusetts Bay Transit Authority.

12. Information Security Software Sales As online attacks persist, spending on information security software continues to rise. The following table gives the forecast for the worldwide sales (in billions of dollars) of information security software through 2007 ($t = 0$ corresponds to 2002):

Year, t	0	1	2	3	4	5
Spending, y	6.8	8.3	9.8	11.3	12.8	14.9

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the spending on information security software in 2008, assuming that the trend continued.

Source: International Data Corporation.

13. Health-Care Spending The following table gives the projected spending on home care and durable medical equipment (in billions of dollars) from 2004 through 2016 ($x = 0$ corresponds to 2004):

Year, x	0	2	4	6	8	10	12
Spending, y	60	74	90	106	118	128	150

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to give the approximate projected spending on home care and durable medical equipment in 2015.
c. Use the result of part (a) to estimate the projected rate of change of the spending on home care and durable medical equipment for the period from 2004 through 2016.

Source: National Association of Home Care and Hospice.

14. IRA Assets The value of all individual retirement accounts (in trillions of dollars) from 2002 through 2005 is summarized in the following table:

Year	2002	2003	2004	2005
Value, y	2.6	3.0	3.3	3.7

a. Letting $x = 2$ denote 2002, find an equation of the least-squares line for these data.
b. Use the results of part (a) to estimate the value of all IRAs in 2006, assuming that the trend continued.
c. Use the result of part (a) to estimate how fast the value of all IRAs was growing over the period from 2002 through 2005.

Source: ici.org.

15. PC Growth The following table gives the projected shipment of personal computers worldwide (in millions of units) from 2004 through 2008 ($x = 4$ corresponds to 2004):

Year, x	4	5	6	7	8
Number, y	174	205	228	253	278

a. Find an equation of the least-squares line for these data.
b. Use the results of part (a) to estimate the shipment of PCs worldwide in 2009, assuming that the projected trend continued.

Source: International Data Corporation.

16. Online Spending The convenience of shopping on the Web combined with high-speed broadband access services is spurring online spending. The projected online spending per buyer (in dollars) from 2002 ($x = 0$) through 2008 ($x = 6$) is given in the following table:

Year, x	0	1	2	3	4	5	6
Spending, y	501	540	585	631	680	728	779

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the rate of change of spending per buyer between 2002 and 2008.

Source: U.S. Department of Commerce.

17. Global Defense Spending The following table gives the projected global defense spending (in trillions of dollars) from the beginning of 2008 ($t = 0$) through 2015 ($t = 7$):

Year, t	0	1	2	3	4	5	6	7
Spending, y	1.38	1.44	1.49	1.56	1.61	1.67	1.74	1.78

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the rate of change in the projected global defense spending from 2008 through 2015.
c. Assuming that the trend continues, what will the global spending on defense be in 2018?

Source: Homeland Security Research.

18. Worldwide Consulting Spending The following table gives the projected worldwide consulting spending (in billions of dollars) from 2005 through 2009 ($x = 5$ corresponds to 2005):

Year, x	5	6	7	8	9
Spending, y	254	279	300	320	345

a. Find an equation of the least-squares line for these data.
b. Use the results of part (a) to estimate the average rate of increase of worldwide consulting spending over the period under consideration.
c. Use the results of part (a) to estimate the amount of spending in 2010, assuming that the trend continued.

Source: Kennedy Information.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 58

19. **Revenue of Moody's Corporation** Moody's Corporation is the holding company for Moody's Investors Service, which has a 40% share in the world credit-rating market. According to *Company Reports,* the total revenue (in billions of dollars) of the company is projected to be as follows ($x = 0$ correspond to 2004):

Year	2004	2005	2006	2007	2008
Revenue, *y*	1.42	1.73	1.98	2.32	2.65

a. Find an equation of the least-squares line for these data.
b. Use the results of part (a) to estimate the rate of change of the revenue of the company for the period in question.
c. Use the result of part (a) to estimate the total revenue of the company in 2010, assuming that the trend continued.

Source: Company Reports.

20. **U.S. Online Banking Households** The following table gives the projected U.S. online banking households as a percentage of all U.S. banking households from 2001 ($x = 1$) through 2007 ($x = 7$):

Year, *x*	1	2	3	4	5	6	7
Percentage of Households, *y*	21.2	26.7	32.2	37.7	43.2	48.7	54.2

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the projected percentage of U.S. online banking households in 2008.

Source: Jupiter Research.

21. **U.S. Outdoor Advertising** U.S. outdoor advertising expenditure (in billions of dollars) from 2002 through 2006 is given in the following table ($x = 0$ correspond to 2002):

Year	2002	2003	2004	2005	2006
Expenditure, *y*	5.3	5.6	5.9	6.4	6.9

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the rate of change of the advertising expenditures for the period in question.

Source: Outdoor Advertising Association.

22. **Online Sales of Used Autos** The amount (in millions of dollars) of used autos sold online in the United States is expected to grow in accordance with the figures given in the following table ($x = 0$ corresponds to 2000):

Year, *x*	0	1	2	3	4	5	6	7
Sales, *y*	1.0	1.4	2.2	2.8	3.6	4.2	5.0	5.8

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the sales of used autos online in 2008, assuming that the predicted trend continued.

Source: comScore Networks, Inc.

23. **Social Security Wage Base** The Social Security (FICA) wage base (in thousands of dollars) from 2004 to 2009 is given in the accompanying table ($x = 1$ corresponds to 2004):

Year	2004	2005	2006	2007	2008	2009
Expenditure, *y*	87.9	90.0	94.2	97.5	102.6	106.8

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the FICA wage base in 2012.

Source: The World Almanac.

24. **Sales of GPS Equipment** The annual sales (in billions of dollars) of global positioning system (GPS) equipment from the year 2000 through 2006 follow ($x = 0$ corresponds to the year 2000):

Year, *x*	0	1	2	3	4	5	6
Annual Sales, *y*	7.9	9.6	11.5	13.3	15.2	16.0	18.8

a. Find an equation of the least-squares line for these data.
b. Use the equation found in part (a) to estimate the annual sales of GPS equipment for 2008, assuming that the trend continued.

Source: ABI Research.

25. **Male Life Expectancy at 65** The Census Bureau projections of male life expectancy at age 65 in the United States are summarized in the following table ($x = 0$ corresponds to 2000):

Year, *x*	0	10	20	30	40	50
Years beyond 65, *y*	15.9	16.8	17.6	18.5	19.3	20.3

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the life expectancy at 65 of a male in 2040. How does this result compare with the given data for that year?
c. Use the result of part (a) to estimate the life expectancy at 65 of a male in 2030.

Source: U.S. Census Bureau.

26. **Authentication Technology** With computer security always a hot-button issue, demand is growing for technology that authenticates and authorizes computer users. The following table gives the authentication software sales (in billions of dollars) from 1999 through 2004 ($x = 0$ represents 1999):

Year, *x*	0	1	2	3	4	5
Sales, *y*	2.4	2.9	3.7	4.5	5.2	6.1

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the sales for 2007, assuming that the trend continued.

Source: International Data Corporation.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 59

27. **CORN USED IN U.S. ETHANOL PRODUCTION** The amount of corn used in the United States for the production of ethanol is expected to rise steadily as the demand for plant-based fuels continue to increase. The following table gives the projected amount of corn (in billions of bushels) used for ethanol production from 2005 through 2010 ($x = 1$ corresponds to 2005):

Year, x	1	2	3	4	5	6
Amount, y	1.4	1.6	1.8	2.1	2.3	2.5

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the amount of corn that will be used for the production of ethanol in 2011, assuming that the trend continued.

Source: U.S. Department of Agriculture.

28. **OPERATIONS MANAGEMENT CONSULTING SPENDING** The following table gives the projected operations management consulting spending (in billions of dollars) from 2005 through 2010 ($x = 5$ corresponds to 2005):

Year, x	5	6	7	8	9	10
Spending, y	40	43.2	47.4	50.5	53.7	56.8

a. Find an equation of the least-squares line for these data.
b. Use the results of part (a) to estimate the average rate of change of operations management consulting spending from 2005 through 2010.
c. Use the results of part (a) to estimate the amount of spending on operations management consulting in 2011, assuming that the trend continued.

Source: Kennedy Information.

In Exercises 29–32, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

29. The least-squares line must pass through at least one data point.

30. The error incurred in approximating n data points using the least-squares linear function is zero if and only if the n data points lie on a nonvertical straight line.

31. If the data consist of two distinct points, then the least-squares line is just the line that passes through the two points.

32. A data point lies on the least-squares line if and only if the vertical distance between the point and the line is equal to zero.

1.5 Solutions to Self-Check Exercises

1. The calculations required for obtaining the normal equations may be summarized as follows:

	x	y	x^2	xy
	1	4	1	4
	3	10	9	30
	4	11	16	44
	5	12	25	60
	7	16	49	112
Sum	20	53	100	250

The normal equations are

$$5b + 20m = 53$$
$$20b + 100m = 250$$

Solving the first equation for b gives

$$b = -4m + \frac{53}{5}$$

which, upon substitution into the second equation, yields

$$20\left(-4m + \frac{53}{5}\right) + 100m = 250$$
$$-80m + 212 + 100m = 250$$
$$20m = 38$$
$$m = 1.9$$

Substituting this value of m into the expression for b found earlier, we find

$$b = -4(1.9) + \frac{53}{5} = 3$$

Therefore, an equation of the least-squares line is

$$y = 1.9x + 3$$

2. The calculations required for obtaining the normal equations may be summarized as follows:

	x	p	x^2	xp
	2.2	38.0	4.84	83.6
	5.4	36.0	29.16	194.4
	7.0	34.5	49.00	241.5
	11.5	30.0	132.25	345.0
	14.6	28.5	213.16	416.1
Sum	40.7	167.0	428.41	1280.6

The normal equations are

$$5b + 40.7m = 167$$
$$40.7b + 428.41m = 1280.6$$

Solving this system of linear equations simultaneously, we find that

$$m \approx -0.81 \quad \text{and} \quad b \approx 40.00$$

Therefore, an equation of the least-squares line is given by

$$p = f(x) = -0.81x + 40$$

which is the required demand equation, provided that

$$0 \le x \le 49.38$$

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 60

USING TECHNOLOGY

Finding an Equation of a Least-Squares Line

Graphing Utility

A graphing utility is especially useful in calculating an equation of the least-squares line for a set of data. We simply enter the given data in the form of lists into the calculator and then use the linear regression function to obtain the coefficients of the required equation.

EXAMPLE 1 Find an equation of the least-squares line for the data

x	1.1	2.3	3.2	4.6	5.8	6.7	8
y	-5.8	-5.1	-4.8	-4.4	-3.7	-3.2	-2.5

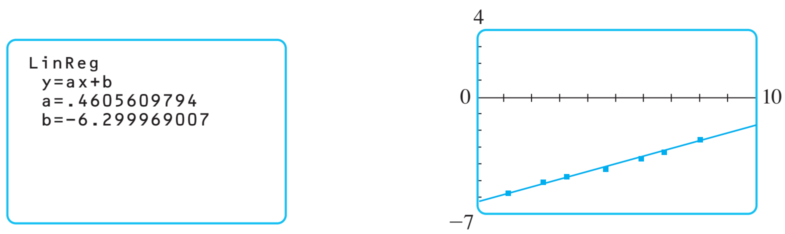
Plot the scatter diagram and the least-squares line for this data.

Solution First, we enter the data as follows:

$x_1 = 1.1$ $y_1 = -5.8$ $x_2 = 2.3$ $y_2 = -5.1$ $x_3 = 3.2$
$y_3 = -4.8$ $x_4 = 4.6$ $y_4 = -4.4$ $x_5 = 5.8$ $y_5 = -3.7$
$x_6 = 6.7$ $y_6 = -3.2$ $x_7 = 8.0$ $y_7 = -2.5$

Then, using the linear regression function from the statistics menu, we obtain the output shown in Figure T1a. Therefore, an equation of the least-squares line ($y = ax + b$) is

$$y = 0.46x - 6.3$$



(a) The TI-83/84 linear regression screen

(b) The scatter diagram and least-squares line for the data

FIGURE **T1**

The graph of the least-squares equation and the scatter diagram for the data are shown in Figure T1b. ■

APPLIED EXAMPLE 2 Demand for Electricity According to Pacific Gas and Electric, the nation's largest utility company, the demand for electricity from 1990 through 2000 is summarized in the following table:

t	0	2	4	6	8	10
y	333	917	1500	2117	2667	3292

Here, $t = 0$ corresponds to 1990, and y gives the amount of electricity demanded in the year t, measured in megawatts. Find an equation of the least-squares line for these data.

Source: Pacific Gas and Electric.

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 61

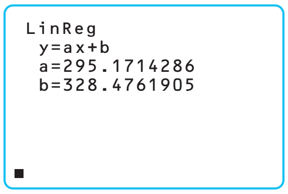


FIGURE **T2**
The TI-83/84 linear regression screen

Solution First, we enter the data as follows:

$x_1 = 0$ $y_1 = 333$ $x_2 = 2$ $y_2 = 917$ $x_3 = 4$ $y_3 = 1500$
$x_4 = 6$ $y_4 = 2117$ $x_5 = 8$ $y_5 = 2667$ $x_6 = 10$ $y_6 = 3292$

Then, using the linear regression function from the statistics menu, we obtain the output shown in Figure T2. Therefore, an equation of the least-squares line is

$$y = 295t + 328$$

■

Excel

Excel can be used to find an equation of the least-squares line for a set of data and to plot a scatter diagram and the least-squares line for the data.

EXAMPLE 3 Find an equation of the least-squares line for the data given in the following table:

x	1.1	2.3	3.2	4.6	5.8	6.7	8.0
y	−5.8	−5.1	−4.8	−4.4	−3.7	−3.2	−2.5

Plot the scatter diagram and the least-squares line for these data.

Solution

1. *Set up a table of values in two columns on a spreadsheet* (Figure T3).
2. *Plot the scatter diagram.* Highlight the numerical values in the table of values. Follow the procedure given in Example 3, page 24, selecting the first chart subtype instead of the second from the **Scatter** chart type. The scatter diagram will appear.
3. *Insert the least-squares line.* Select the **Layout** tab, and click on **Trendline** in the **Analysis** group. Next, click on **More Trendline Options...** in the same subgroup. In the **Format Trendline** dialog box that appears, click on **Display Equation on chart**.

$$y = 0.4606x - 6.3$$

and the least-squares line will appear on the chart (Figure T4).

	A	B
1	x	y
2	1.1	-5.8
3	2.3	-5.1
4	3.2	-4.8
5	4.6	-4.4
6	5.8	-3.7
7	6.7	-3.2
8	8	-2.5

FIGURE **T3**
Table of values for *x* and *y*

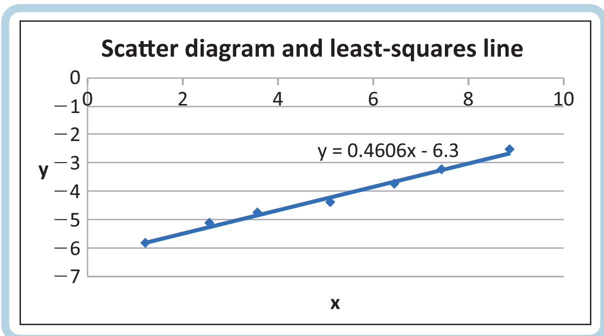


FIGURE **T4**
Scatter diagram and least-squares line for the given data

■

The following example requires the Analysis ToolPak. Use Excel's Help function to learn how to install this add-in.

Note: Boldfaced words/characters enclosed in a box (for example, **Enter**) indicate that an action (click, select, or press) is required. Words/characters printed blue (for example, Chart sub-type:) indicate words/characters that appear on the screen. Words/characters printed in a monospace font (for example, `=(−2/3)*A2+2`) indicate words/characters that need to be typed and entered.

(*continued*)

48149_01_ch1_p001-066 11/2/10 10:35 AM Page 62

APPLIED EXAMPLE 4 Demand for Electricity According to Pacific Gas and Electric, the nation's largest utility company, the demand for electricity from 1990 through 2000 is summarized in the following table:

t	0	2	4	6	8	10
y	333	917	1500	2117	2667	3292

Here, $t = 0$ corresponds to 1990, and y gives the amount of electricity in year t, measured in megawatts. Find an equation of the least-squares line for these data.

Solution

1. *Set up a table of values on a spreadsheet* (Figure T5).
2. *Find the equation of the least-squares line for the data.* Click on **Data Analysis** in the Analysis group of the Data tab. In the Data Analysis dialog box that appears, select **Regression**, and then click **OK**. In the Regression dialog box that appears, select the **Input Y Range:** box, and then enter the y-values by highlighting cells B2:B7. Next select the **Input X Range:** box, and enter the x-values by highlighting cells A2:A7. Click **OK**, and a **SUMMARY OUTPUT** worksheet will appear. In the third table, you will see the entries shown in Figure T6. These entries give the value of the y-intercept and the coefficient of x in the equation $y = mx + b$. In our example, we are using the variable t instead of x, so the required equation is

$$y = 295t + 328$$

	A	B
1	t	y
2	0	333
3	2	917
4	4	1500
5	6	2117
6	8	2667
7	10	3292

FIGURE **T5**
Table of values for x and y

	Coefficients
Intercept	328.4761905
X Variable 1	295.1714286

FIGURE **T6**
Entries in the SUMMARY OUTPUT box

TECHNOLOGY EXERCISES

In Exercises 1–4, find an equation of the least-squares line for the given data.

1.

x	2.1	3.4	4.7	5.6	6.8	7.2
y	8.8	12.1	14.8	16.9	19.8	21.1

2.

x	1.1	2.4	3.2	4.7	5.6	7.2
y	−0.5	1.2	2.4	4.4	5.7	8.1

3.

x	−2.1	−1.1	0.1	1.4	2.5	4.2	5.1
y	6.2	4.7	3.5	1.9	0.4	−1.4	−2.5

4.

x	−1.12	0.1	1.24	2.76	4.21	6.82
y	7.61	4.9	2.74	−0.47	−3.51	−8.94

5. MODELING WITH DATA According to *Company Reports,* Starbucks annual sales (in billions of dollars) for 2001 through 2006 are as follows ($x = 0$ corresponds to 2001):

Year, x	0	1	2	3	4	5
Sales, y	2.65	3.29	4.08	5.29	6.37	7.79

a. Find an equation of the least-squares line for these data.
b. Use the result to estimate Starbucks sales for 2009, assuming that the trend continued.

Source: Company Reports.

6. MODELING WITH DATA The projected number of computers (in millions) connected to the Internet in Europe from 1998 through 2002 is summarized in the following table ($x = 0$ corresponds to the beginning of 1998):

Year, x	0	1	2	3	4
Net-Connected Computers, y	21.7	32.1	45.0	58.3	69.6

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the projected number of computers connected to the Internet in Europe at the beginning of 2005, assuming that the trend continued.

Source: Dataquest, Inc.

7. **Modeling With Data** According to data from the Council on Environmental Quality, the amount of waste (in millions of tons per year) generated in the United States from 1960 to 1990 was as follows:

Year	1960	1965	1970	1975
Amount, y	81	100	120	124

Year	1980	1985	1990
Amount, y	140	152	164

(Let x be in units of 5 and let $x = 1$ represent 1960.)

a. Find an equation of the least-squares line for these data.

b. Use the result of part (a) to estimate the amount of waste generated in the year 2000, assuming that the trend continued.

Source: Council on Environmental Quality.

8. **Modeling With Data** More and more travelers are purchasing their tickets online. According to industry projections, the U.S. online travel revenue (in billions of dollars) from 2001 through 2005 is given in the following table ($t = 0$ corresponds to 2001):

Year, t	0	1	2	3	4
Revenue, y	16.3	21.0	25.0	28.8	32.7

a. Find an equation of the least-squares line for these data.

b. Use the result of part (a) to estimate the U.S. online travel revenue for 2006.

Source: Forrester Research, Inc.

9. **Modeling With Data** According to the U.S. Department of Energy, the consumption of energy by countries of the industrialized world is projected to rise over the next 25 years. The consumption (in trillions of cubic feet) from 2000 through 2025 is summarized in the following table:

Year	2000	2005	2010	2015	2020	2025
Consumption, y	214	225	240	255	270	285

(Let x be measured in 5-year intervals, and let $x = 0$ represent 2000.)

a. Find an equation of the least-squares line for these data.

b. Use the result of part (a) to estimate the energy consumption in the industrialized world in 2012.

Source: U.S. Department of Energy.

10. **Modeling With Data** With an aging population, the demand for health care, as measured by outpatient visits, is steadily growing. The number of outpatient visits (in millions) from 1991 through 2001 is recorded in the following table ($x = 0$ corresponds to 1991):

Year, x	0	1	2	3	4	5	6	7	8	9	10
Number of Visits, y	320	340	362	380	416	440	444	470	495	520	530

a. Find an equation of the least-squares line for these data.

b. Use the result of part (a) to estimate the number of outpatient visits in 2004, assuming that the trend continued.

Source: PriceWaterhouse Coopers.

CHAPTER 1 Summary of Principal Formulas and Terms

FORMULAS

1. Distance between two points	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. Equation of a circle	$(x - h)^2 + (y - k)^2 = r^2$
3. Slope of a nonvertical line	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
4. Equation of a vertical line	$x = a$
5. Equation of a horizontal line	$y = b$
6. Point-slope form of the equation of a line	$y - y_1 = m(x - x_1)$
7. Slope-intercept form of the equation of a line	$y = mx + b$
8. General equation of a line	$Ax + By + C = 0$

TERMS

Cartesian coordinate system (2)	dependent variable (28)	demand function (31)
ordered pair (2)	domain (28)	supply function (32)
coordinates (3)	range (28)	break-even point (41)
parallel lines (12)	linear function (28)	market equilibrium (43)
perpendicular lines (14)	total cost function (30)	equilibrium quantity (43)
function (28)	revenue function (30)	equilibrium price (43)
independent variable (28)	profit function (30)	

CHAPTER 1 Concept Review Questions

Fill in the blanks.

1. A point in the plane can be represented uniquely by a/an _____ pair of numbers. The first number of the pair is called the _____, and the second number of the pair is called the _____.

2. a. The point $P(a, 0)$ lies on the _____ axis, and the point $P(0, b)$ lies on the _____ axis.
b. If the point $P(a, b)$ lies in the fourth quadrant, then the point $P(-a, b)$ lies in the _____ quadrant.

3. The distance between two points $P(a, b)$ and $P(c, d)$ is _____.

4. An equation of a circle with center $C(a, b)$ and radius r is given by _____.

5. a. If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are any two distinct points on a nonvertical line L, then the slope of L is $m =$ _____.
b. The slope of a vertical line is _____.
c. The slope of a horizontal line is _____.
d. The slope of a line that slants upward is _____.

6. If L_1 and L_2 are distinct nonvertical lines with slopes m_1 and m_2, respectively, then: L_1 is parallel to L_2 if and only if _____; and L_1 is perpendicular to L_2 if and only if _____.

7. a. An equation of the line passing through the point $P(x_1, y_1)$ and having slope m is _____. It is called the _____ form of an equation of a line.
b. An equation of the line that has slope m and y-intercept b is _____. It is called the _____ form of an equation of a line.

8. a. The general form of an equation of a line is _____.
b. If a line has equation $ax + by + c = 0$ $(b \neq 0)$, then its slope is _____.

9. A linear function is a function of the form $f(x) =$ _____.

10. a. A demand function expresses the relationship between the unit _____ and the quantity _____ of a commodity. The graph of the demand function is called the _____ curve.
b. A supply function expresses the relationship between the unit _____ and the quantity _____ of a commodity. The graph of the supply function is called the _____ curve.

11. If $R(x)$ and $C(x)$ denote the total revenue and the total cost incurred in manufacturing x units of a commodity, then the solution of the simultaneous equations $y = C(x)$ and $y = R(x)$ gives the _____ point.

12. The equilibrium quantity and the equilibrium price are found by solving the system composed of the _____ equation and the _____ equation.

CHAPTER 1 Review Exercises

In Exercises 1–4, find the distance between the two points.

1. (2, 1) and (6, 4) **2.** (9, 6) and (6, 2)

3. $(-2, -3)$ and $(1, -7)$ **4.** $\left(\frac{1}{2}, \sqrt{3}\right)$ and $\left(-\frac{1}{2}, 2\sqrt{3}\right)$

In Exercises 5–10, find an equation of the line L that passes through the point $(-2, 4)$ and satisfies the given condition.

5. L is a vertical line. **6.** L is a horizontal line.

7. L passes through the point $\left(3, \frac{7}{2}\right)$.

8. The x-intercept of L is 3.

9. L is parallel to the line $5x - 2y = 6$.

10. L is perpendicular to the line $4x + 3y = 6$.

11. Find an equation of the line with slope $-\frac{1}{2}$ and y-intercept -3.

12. Find the slope and y-intercept of the line with equation $3x - 5y = 6$.

13. Find an equation of the line passing through the point (2, 3) and parallel to the line with equation $3x + 4y - 8 = 0$.

14. Find an equation of the line passing through the point $(-1, 3)$ and parallel to the line joining the points $(-3, 4)$ and (2, 1).

15. Find an equation of the line passing through the point $(-2, -4)$ that is perpendicular to the line with equation $2x - 3y - 24 = 0$.

In Exercises 16 and 17, sketch the graph of the equation.

16. $3x - 4y = 24$ **17.** $-2x + 5y = 15$

18. Sales of MP3 Players Sales of a certain brand of MP3 players are approximated by the relationship

$$S(x) = 6000x + 30{,}000 \qquad (0 \leq x \leq 5)$$

where $S(x)$ denotes the number of MP3 players sold in year x ($x = 0$ corresponds to the year 2007). Find the number of MP3 players expected to be sold in 2012.

19. Company Sales A company's total sales (in millions of dollars) are approximately linear as a function of time (in years). Sales in 2006 were \$2.4 million, whereas sales in 2011 amounted to \$7.4 million.
a. Letting $x = 0$ correspond to 2006, find a function giving the company's sales in terms of x.
b. What were the sales in 2009?

20. Show that the triangle with vertices $A(1, 1)$, $B(5, 3)$, and $C(4, 5)$ is a right triangle.

21. Clark's Rule Clark's Rule is a method for calculating pediatric drug dosages based on a child's weight. If a denotes the adult dosage (in milligrams) and if w is the child's weight (in pounds), then the child's dosage is given by

$$D(w) = \frac{aw}{150}$$

a. Show that D is a linear function of w.
b. If the adult dose of a substance is 500 mg, how much should a 35-lb child receive?

22. Linear Depreciation An office building worth \$6 million when it was completed in 2010 is being depreciated linearly over 30 years with a scrap value of \$0.
a. What is the rate of depreciation?
b. What will be the book value of the building in 2020?

23. Linear Depreciation In 2006 a manufacturer installed a new machine in her factory at a cost of \$300,000. The machine is depreciated linearly over 12 years with a scrap value of \$30,000.
a. What is the rate of depreciation of the machine per year?
b. Find an expression for the book value of the machine in year t ($0 \leq t \leq 12$).

24. Profit Functions A company has a fixed cost of \$30,000 and a production cost of \$6 for each disposable camera it manufactures. Each camera sells for \$10.
a. What is the cost function?
b. What is the revenue function?
c. What is the profit function?
d. Compute the profit (loss) corresponding to production levels of 6000, 8000, and 12,000 units, respectively.

25. Demand Equations There is no demand for a certain commodity when the unit price is \$200 or more, but the demand increases by 200 units for each \$10 decrease in price below \$200. Find the demand equation and sketch its graph.

26. Supply Equations Bicycle suppliers will make 200 bicycles available in the market per month when the unit price is \$50 and 2000 bicycles available per month when the unit price is \$100. Find the supply equation if it is known to be linear.

In Exercises 27 and 28, find the point of intersection of the lines with the given equations.

27. $3x + 4y = -6$ and $2x + 5y = -11$

28. $y = \frac{3}{4}x + 6$ and $3x - 2y + 3 = 0$

29. Break-Even Analysis The cost function and the revenue function for a certain firm are given by $C(x) = 12x + 20{,}000$ and $R(x) = 20x$, respectively. Find the break-even point for the company.

30. Market Equilibrium Given the demand equation $3x + p - 40 = 0$ and the supply equation $2x - p + 10 = 0$, where p is the unit price in dollars and x represents the quantity demanded in units of a thousand, determine the equilibrium quantity and the equilibrium price.

31. College Admissions The accompanying data were compiled by the Admissions Office of Carter College during the past 5 years. The data relate the number of college brochures and follow-up letters (x) sent to a preselected list of high school juniors who took the PSAT and the number of completed applications (y) received from these students (both measured in thousands).

Brochures Sent, x	1.8	2	3.2	4	4.8
Applications Completed, y	0.4	0.5	0.7	1	1.3

a. Derive an equation of the straight line L that passes through the points (2, 0.5) and (4, 1).
b. Use this equation to predict the number of completed applications that might be expected if 6400 brochures and follow-up letters are sent out during the next year.

48149_01_ch1_p001-066 11/16/10 9:26 AM Page 66

32. **Market Equilibrium** The demand equation for the Edmund compact refrigerator is $2x + 7p - 1760 = 0$, where x is the quantity demanded each week and p is the unit price in dollars. The supply equation for these refrigerators is $3x - 56p + 2680 = 0$, where x is the quantity the supplier will make available in the market when the wholesale price is p dollars each. Find the equilibrium quantity and the equilibrium price for the Edmund compact refrigerators.

33. **Modeling With Data** The average hourly salary (in dollars) of hospital nurses in metropolitan Boston from 2000 through 2004 is given in the following table ($x = 0$ corresponds to 2000):

Year	2000	2001	2002	2003	2004
Hourly Salary, *y*	27	29	31	32	35

a. Find an equation of the least-squares line for these data.
b. If the trend continued, what was the average hourly salary of nurses in 2006?

Source: American Association of Colleges of Nursing.

34. **Modeling With Data** The Census Bureau projections of female life expectancy at age 65 in the United States are summarized in the following table ($x = 0$ corresponds to 2000):

Year, *x*	0	10	20	30	40	50
Years beyond 65, *y*	19.5	20.0	20.6	21.2	21.8	22.4

a. Find an equation of the least-squares line for these data.
b. Use the result of part (a) to estimate the life expectancy at 65 of a female in 2040. How does this result compare with the given data for that year?
c. Use the result of part (a) to estimate the life expectancy at 65 of a female in 2030.

Source: U.S. Census Bureau.

The problem-solving skills that you learn in each chapter are building blocks for the rest of the course. Therefore, it is a good idea to make sure that you have mastered these skills before moving on to the next chapter. The Before Moving On exercises that follow are designed for that purpose. After taking this test, you can see where your weaknesses, if any, are. Then you can log in at login.cengagebrain.com where you will find a link to CourseMate. Here, you can access additional Concept Quiz and Concept Review modules.

If you feel that you need additional help with these exercises, at this Website you can also use Tutorial Example Videos.

CHAPTER 1 Before Moving On . . .

1. Plot the points $A(-2, 1)$ and $B(3, 4)$ on the same set of axes, and find the distance between A and B.

2. Find an equation of the line passing through the point (3, 1) and parallel to the line $3x - y - 4 = 0$.

3. Let L be the line passing through the points (1, 2) and (3, 5). Is L perpendicular to the line $2x + 3y = 10$?

4. The monthly total revenue function and total cost function for a company are $R(x) = 18x$ and $C(x) = 15x + 22{,}000$, respectively, where x is the number of units produced and both $R(x)$ and $C(x)$ are measured in dollars.
   a. What is the unit cost for producing the product?
   b. What is the monthly fixed cost for the company?
   c. What is the selling price for each unit of the product?

5. Find the point of intersection of the lines $2x - 3y = -2$ and $9x + 12y = 25$.

6. The annual sales of Best Furniture Store are expected to be given by $S_1 = 4.2 + 0.4t$ million dollars t years from now, whereas the annual sales of Lowe's Furniture Store are expected to be given by $S_2 = 2.2 + 0.8t$ million dollars t years from now. When will Lowe's annual sales first surpass Best's annual sales?