Name _____ School _____ Date _____

Lab 31.1 – Radioactive Half-Life

Purpose
To study the operation of the Geiger tube
To investigate the Geiger tube's response to changes in tube voltage
To learn to measure background radiation
To investigate the change in activity of a radioactive isotope over time
To learn to use the technique of elution of a short half-life isotope from an isotope with a longer half-life
To determine the half-life of an isotope graphically

Equipment
Virtual Geiger Counter
PENCIL

Explore the Apparatus/Theory
Open the Virtual Geiger Counter lab. Roll your pointer over the various parts of the apparatus and read the information that pops up in the text box. Be sure to note the animation that runs when you roll over the Geiger tube.
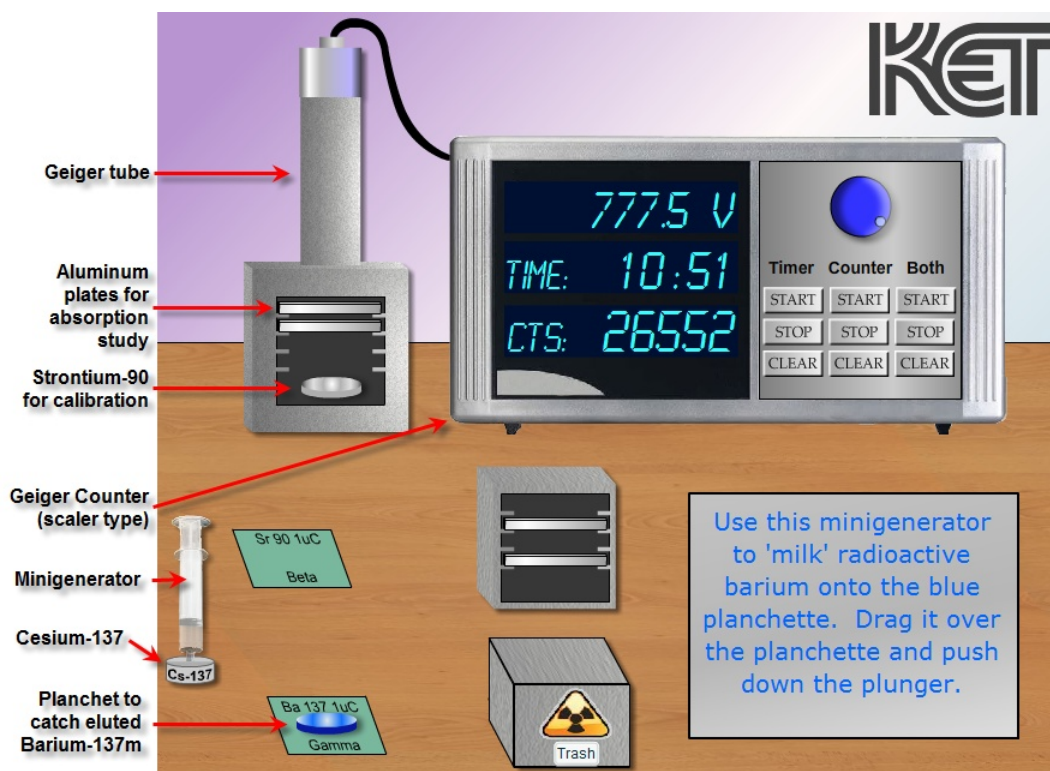

Figure 1
This investigation is divided into two parts.

- In the first part you'll learn to use a Geiger counter to detect and count radiation.
  Before you can get an accurate count you'll first need to find and set the operating voltage of the Geiger tube.

- In the second part of the lab you'll use a minigenerator to wash a short half-life isotope of Barium from a sample of Cesium. You'll measure the activity of the Barium as a function of time and use your data to determine the half-life of the Barium isotope.

1. Use of the Geiger counter for detecting and counting radiation

In this lab you'll observe the radiation produced by Strontium-90 and Barium-137m. Both of these are gamma emitters. The instrument you'll use, the Geiger tube, is capable of detecting all three types of nuclear radiation – alpha (α), beta (β), and gamma (γ), provided they have sufficient energy.

In Figure 2 a frame of animation is superimposed on the Geiger tube. In the lab roll your pointer over the tube to view the full animation of the tube in operation.

The **sample box** at the bottom of the apparatus holds any sample that needs to be analyzed. The radiation goes in all directions but a fraction of it will pass through the thin, fragile mica window at the bottom of the tube. **Radiation of sufficient energy will ionize the argon gas in the tube.** Argon gas molecules are represented by the pairs of + & - charges. Note that the one at the lower right has just been ionized.

A potential difference is maintained between the walls of the cylindrical tube containing the gas and a wire running along the axis of the tube. Electrons freed in the ionization are attracted to the central **anode**. Positive ions are attracted to the cylinder, the **cathode**. If the voltage between the wire and cylinder is large enough a freed electron will have sufficient energy to ionize another argon atom before arriving at the anode. These two electrons can ionize two more atoms resulting in four freed electrons, and then eight, sixteen, etc.

If large enough, this **cascade** of electrons produces a very brief but measureable current in the external circuit connected between the anode and cathode. This has two results.

- A small amount of this current is sent to the counter to register the event by incrementing the number of **counts** on the Geiger counter display.

- The potential difference between the wire and cylinder is reduced enough by this discharge that for a brief interval no further discharge occurs.
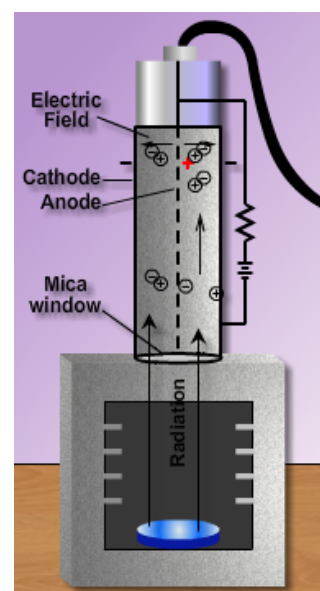

Figure 2

After a few microseconds the voltage is restored by the battery and another event can be measured. If this **dead time** is too large, some radiation entering the tube will go undetected. Keeping the voltage high enough reduces this dead time but if the voltage is too high, it can damage the tube. You'll learn how to determine the optimal operating voltage next.

Finding the Geiger Tube Operating Voltage

With no voltage between the electrodes, the ions and electrons will not migrate to the cathode and anode. Thus no current flows and no counts are registered. As the voltage is increased, at some minimum **threshold voltage** counts will begin to be registered. This voltage provides just enough energy for the most energetic radiation to cause a cascade.

As the voltage is increased the count rate increases at a fairly linear rate. This is due to the variation in energy of the radiation. The more energetic it is the less voltage is needed to cause a cascade. As the voltage is increased, cascades can be started by increasingly less energetic radiation.

Eventually a **plateau** is reached where the count rate is approximately independent of the voltage. A more professional grade counter will have an almost flat plateau. Of course we can only afford the cheap student model, so ours has a slight upward slope.

Beyond this plateau is another steep region where the count rate again increases with voltage. In this region the voltage is high enough to cause continuous discharges in the gas which don't indicate actual counts of radiation and which can damage the tube.

The **operating voltage** is not a fixed value, but a good choice is somewhat past the beginning of the plateau. Different models of Geiger tubes have different threshold voltages so the first thing that must be done when using a Geiger tube is to determine its operating voltage. You'll do that by measuring the **count rate** for a sample of Strontium-90 at a range of voltages and plot the result – count rate vs. voltage.

If you had some problem picturing what was described above, don't worry, you'll see the real thing now. Be sure and look back at the highlighted terms above after you create your graph. You should be able to identify them on your graph.

Procedure

1. In Grapher (or Excel if you prefer) create an empty data table with Voltage (V) and Counts (no units) as your first two columns of data.

2. You're going to plot counts/min vs. voltage. To get counts/ min you'll record counts for 30 seconds and then multiply the number of counts by 2. Logger Pro can do that repetitive calculation for you automatically. Let's set that up now. Add a third column by using "Data/New Calculated Column." Name this column Count Rate, abbreviated CR, and with units min^{-1}. For the equation you'll want "Counts"*2. If you've forgotten how to do this, consult your lab manual.

Set the axes to run from 0 to 60,000 (counts per minute, (min^{-1})) on the ordinate and 0 to 800 (Volts) on the abscissa. Right click in the graph and select "Graph Options." Turn on point protectors and turn off "Connect Points." Title the graph "Count Rate vs. Voltage – Strontium-90."

As you take data, enter your values directly into the Grapher (or Excel) table. This will give you a better idea of how the count rate depends on the voltage than if you took the data and plotted it later. It will also keep you from getting the voltage so high that you (virtually) damage the tube.

3. You'll use a Strontium-90 gamma source for this test. Drag and drop it into the sample box below the tube.

4. **You now want to find the threshold voltage, the minimum voltage that will produce any counts with this tube.** Using the blue voltage knob, set the voltage to zero volts.

There are three columns of buttons on the counter console. The first column controls the **timer** which records in minutes and seconds. The second controls the **counter**. The third column controls both the **timer** and **counter** simultaneously. For the process of finding the threshold voltage you just need to look for counts, so you'll just use the column of counter buttons.

5. Start the counter by clicking its **Counter Start** button. Nothing should happen since the voltage is zero. Turn the blue voltage knob to gradually increase the voltage until you first begin to see counts. This knob works like other rotatable objects in these labs. That is, you can click on the knob handle and then drag away from the knob and then drag around in a large circle, thus giving you greater resolution. Go ahead and find the threshold voltage. It should be somewhere above 300 V. You can increase and decrease the voltage and watch the count start and stop, thus homing in on the threshold voltage.

Record your threshold voltage as the first entry in the voltage column in your Grapher (or Excel) data table.

6. You now want to measure the number of counts per minute for the threshold voltage. Make your reading over 30 seconds. To ensure that the timer and counter start together you'll use the "**Both**" column of buttons. Here's how.

Clear everything using the **Clear Both** button. Then **Start Both**. As soon as the timer reaches 30 seconds, **Stop Both**. Note that our apparatus only displays the number of counts at the end of each second rather than displaying each count as it comes in. (We hope to be able to afford a better counter some day.) You'll note that the counts values changes just after the seconds value, so be sure to click Stop after the counts have changed.

Record the number of counts beside your threshold voltage in the data table. The spread sheet should automatically fill in the count rate value. This should be between 100 and 200.

Clear Both to clear both readings. You'll always need to do this between trials.

7. Increase the voltage by about 25 volts and make another 30-second count. Repeat this process in approximately 25-volt increments of voltage until you see the graph begin to level off. On the graph, drag across all the points in the rising sequence. (Probably three or four.) Click "Linear Fit." You should get a steep line.

8. You're now on the plateau. Continue taking and recording 30-second count readings until you see a sharp jump in the counts per minute. Take two additional readings beyond that point.

9. You should now have data indicating a total of three straight lines. Select all the points in the plateau and create the line of best fit. Do the same for the steep line at the end of your data. The graph should look much like a chair facing left.

10. The operating voltage that you use for the rest of the lab should be chosen at about 100 volts into the plateau. Choose a voltage in that region and record it below. Use this voltage for all subsequent measurements.

Operating Voltage _____ V

Measuring the Background Radiation

There is no escaping some amount of radiation in our environment – building materials, cosmic radiation, etc. Even our bodies emit some. If this background radiation is significant you'll need to subtract it from all your future measurements.

1. Remove the Strontium-90 sample from the chamber and drop it back on its pad. We'll assume that it and the Barium-137 sources are both far enough away from the Geiger tube so as to provide no measureable radiation. To measure the background radiation, set your voltage at your selected operating voltage and take a four-minute reading. Calculate and record your background radiation below in counts per minute.

Background Radiation _____ counts/min

2. Radioactive Half-Life

Theory

The distinguishing characteristic of radioactive nuclei is their tendency to spontaneously decay over time. The probability of decay during a given amount of time is the same for all nuclei of a given isotope, but differs over a wide range from one isotope to another. Techniques such as radioactive dating rely on this predictable nature of isotopes.

While each individual nucleus of a given isotope has the same probability of decay per second, the actual time for a given nucleus to decay is completely unpredictable. But for a substantially large sample, the average behavior is quite predictable. A loose analogy is the popping of popcorn. While the time it takes for a specific kernel in the bag to pop is unpredictable, we can still program a button on the microwave for a set time to pop a bag and get good results. An average popcorn kernel lifetime is pretty consistent. Actually a specific strain of popcorn will vary in its behavior depending on such factors as moisture content and age. Radioactive nuclei are unaffected by any such influences.

With our radioactive nuclei the **decay constant, λ**, specifies the probability of decay per time for one nucleus. It has units of s^{-1}, min^{-1}, etc. A nucleus of an isotope with a decay constant of $100 \ s^{-1}$ would be twenty times more likely to decay during a given second than one with a decay constant of $5 \ s^{-1}$.

If we have N nuclei of an isotope with a decay constant of λ, the decay rate of these nuclei, $\Delta N/\Delta t$ is given by

$$\frac{\Delta N}{\Delta t} = -\lambda N \qquad (1)$$

In words this means that the rate of decay, $\frac{\Delta N}{\Delta t}$, of N nuclei equals the probability of decay of one nuclei, λ, times the number of nuclei, N. The minus sign indicates that the number of nuclei is decreasing. That is, if N_o is the number of nuclei at some time and N is the number sometime later, $\Delta N = N - N_o$, is negative.

We can do a little calculus with equation one to create an equation relating N and N_o. Don't panic. No calculus is required in this course. The result is

$$N = N_o e^{-\lambda t} \qquad (2)$$

The value of $e^{-\lambda t}$ has a value with a range limited to $0 < e^{-\lambda t} \le 1$. The larger the value of t, the smaller the value of $e^{-\lambda t}$. When multiplied by N_o, it gives a value for N that decreases exponentially over time. Your next graph will be of this form.

This equation is not particularly useful to us in this lab since we have no way of measuring the number of nuclei. But with a little more mathematical effort we can show that the activity, A, which we can measure, varies in the same way as N.

The **activity, A**, (decays/time^{-1}, e.g. s^{-1}, min^{-1}) is the absolute value of the decay rate.

$$A = \lambda N \qquad (3)$$

With a little more calculus, we find

$$A = A_o e^{-\lambda t} \qquad (4)$$

As before, this tells us that the activity, A, of N nuclei decreases exponentially over time. From equation 3 we see that the activity depends both on the decay constant, λ, and the number of nuclei present, N.

Going one step further we can derive the value for another quantity, the half-life, $t_{1/2}$. Dividing both sides by A_o, we get

$$\frac{A}{A_o} = e^{-\lambda t} \qquad (5)$$

Inverting both sides and then taking the natural log of both sides, we find

$$\ln\left(\frac{A_o}{A}\right) = \lambda t \tag{6}$$

We define the **half-life, $t_{1/2}$**, as the time for the activity, A, or number of nuclei, N, to halve. Thus

$$\ln\left(\frac{A_o}{A_{0/2}}\right) = \ln(2) = \lambda\, t_{1/2} \tag{7}$$

Substituting .693 for $\ln(2)$ and solving for $t_{1/2}$, we find

$$\boldsymbol{t_{1/2}} = \frac{.693}{\lambda} \tag{8}$$

A note about N

Any description of probabilistic behavior is only meaningful for large numbers of events. When you flip a coin you know that you have a fifty-fifty chance of a heads coming up. So after several tosses you might expect equal numbers of heads and tails. But if you tried 10 tosses, you might very well find 7 heads and 3 tails. But with 100 tosses you'd usually get something closer to 50 of each. And with 1000, it looks even better. The larger the numbers, the better the results match the probabilities. So with your investigation of radioactivity you'll always want to work with large numbers of decays.

The Minigenerator

When you were determining the operating voltage for the Geiger tube it would have been difficult to make any determination without a steady decay rate. Otherwise you wouldn't know if a change in count rate was related to the voltage or the actual activity of our sample. For this reason we chose Strontium-90 which has a half-life of 28.1 years. With a half-life that long, N and thus A, would be essentially constant during our investigation.

Now for our investigation of half-lives we need an isotope that will change its activity substantially during a reasonable time. Otherwise we wouldn't observe any change in activity during our investigation. This leads to a problem. Any isotope with a short enough half-life is going to decay so quickly that N will be too small to be useful in a pretty short time.

This brings up the question of where isotopes with short half-lives come from. Shouldn't any that ever existed be pretty-well gone? The isotope that you'll be using both answers this question and solves the problem in the previous paragraph.

Our lab uses a simple but very effective tool called a Minigenerator^{TM} developed by Union Carbide. The system uses a sample of Cesium-137 which beta decays with a half-life of 30.1 years to an **isomeric** (excited) **state** of Barium-137. The Barium-137m gamma decays with a short half-life to a stable form of Barium-137. (The m or an * indicates an excited or metastable state.)

$$^{137}_{55}Cs \rightarrow {}^{137}_{56}Ba^* + {}^{0}_{-1}e$$
$$\downarrow$$
$$^{137}_{56}Ba + \gamma$$
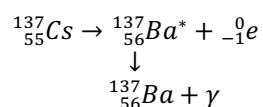
The Cesium-137 is held in a small container affixed to the bottom of a syringe containing a hydrochloric acid-saline solution. When the acid-saline solution is forced through the Cs-137, the Barium-137m is **eluted** (washed out) and into a small cup called a **planchette**. (Blue disk.)
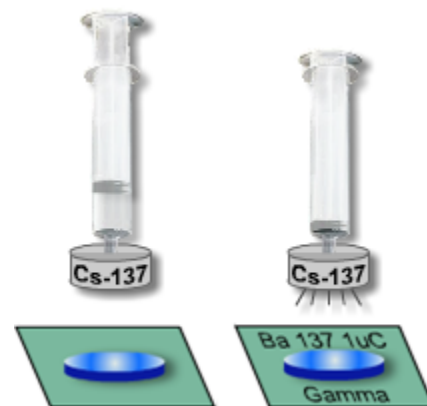

Figure 3

This process is sometimes referred to as "milking." The daughter product Ba-137m is milked from the parent isotope, Cs-137. This milking can be repeated many times, but approximately 12 min is required for the Ba-137m to build back up by the decay of more Cs-137. We initially planned to include this forced waiting period in this virtual lab, but the alert student could circumvent it by just restarting the lab.

But you will find that if, sometime after milking, you want to start again, you'll need a new planchette. Otherwise you'd be mixing two samples with different activities. (Study some nuclear engineering if you want to see what a nightmare that can be.) If you place the used planchette on the pad and click the "Trash" button, the planchette will be replaced. You'll also note a nod to the law of conservation of momentum when you do this.
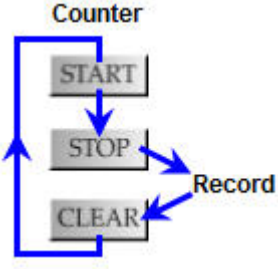
Procedure

Data Collection

Steps 1-4 of this procedure are time-sensitive. That is, once you start, you have to keep moving, especially once you begin step 4. If you get confused in step 4 and need to pause, you'll have to start again. (See "Trash" above.) So the best idea is to commit the following procedure to memory by doing some mental test runs. This procedure looks as easy as riding a bicycle. Be prepared to fall off!

1) Set the voltage of your Geiger counter to (approximately) the operating voltage you determined earlier. Record this value in Table 1. Also record the background radiation value that you measured earlier.

2) Elute your Barium-137m sample. Here's how. Drag the syringe and release it over the planchette. It should snap in place. Click and drag the plunger downward and then release it. This should work very realistically. You can only do this once. Remember, if you need to restart, press the trash button to get a new planchette.

3) Drag your planchette into the chamber below the Geiger tube.

4) Here's the one-sentence version of the data collection procedure that follows:

Determine the number of decays of the Ba-137m sample for a 15-second interval. Repeat eleven times beginning at the start of each successive minute.
Here's how to do it:

a)	Click "**Start Both**" to start both the timer and the counter. You'll leave the timer running for the rest of the data taking.	Both START
	You'll only use the counter buttons from here on.	Counter
b)	At the end of 15 seconds, click "**Stop Counter**." The counter display shows the number of counts during the first 15-second interval. **Record** that value in your data table under "N counts." Then "**Clear Counter**."	START STOP CLEAR Record
c)	When the timer reads 1:00 (1 minute, zero seconds), "**Start Counter**."	
d)	At 1:15, "**Stop Counter**" and **record** the number of counts. "**Clear Counter**."	
e)	Repeat c) and d) at the beginning of each minute until you've recorded the value for 12:00 – 12:15.	

Data Analysis

You now have thirteen (t, N) data pairs. You have several calculations that need to be made to each piece of data. You really don't want to do all this by hand. You can use equations to do these calculations for you if you wish.

5) Transfer your "Time at start of interval" and "Counts in 15 sec" values to Grapher.

6) You now need to **convert each N value to activity in counts/min**. To each of these values you need to **subtract the background radiation**. Let's do that in one step.

   - Create a new calculated data column. (Data/New Calculated Column.)

   - Name this column "Activity" with units of min^{-1}.

   - For the equation, you need to multiply "Counts" by 4 and subtract your background radiation from that result.

     Experiment with that until you come up with something that's successful.
     Even though this is the average activity for a 15-second period we'll associate it with the beginning instant of the time interval. So the first activity will be associated with 0.0 min, etc.

7) Plot activity vs. time. You should get a classic exponential decay graph.

8) From your graph make a rough determination of the half-life by finding the time for the activity to decrease from A_o to $\frac{1}{2}A_o$ and from $\frac{1}{2}A_o$ to $\frac{1}{4}A_o$. This is a bit tricky. Record these values on your data sheet.

9) Calculate and record the average of these two values as your average experimental half-life.

10) Determine the percent error in your experimental half-life using the accepted value of 2.6 minutes.

11) Determine the experimental decay constant, λ, from your average experimental half-life using Equation 8.

12) The decay constant can also be found graphically as follows. We'll start with equation 4.

$$A = A_o e^{-\lambda t} \tag{4}$$

Taking the natural log of both sides and realizing that the log of a product is the sum of the logs, we get

$$\ln A = \ln A_o e^{-\lambda t} = \ln(e^{-\lambda t}) + \ln A_o \tag{9}$$

$$\boldsymbol{\ln A = -\lambda t + \ln A_o} \tag{10}$$

Equation (10) is in the form y = mx + b, which describes a straight line where

ln A is the ordinate,

ln A₀ is the y-intercept

t is the abscissa, and

-λ is the slope.

Plotting a graph of ***ln A* vs *t*** will allow you experimentally determine the activity, λ graphically. You can plot this line on the same graph as before by using the right Y-axis for the ln A values as explained below.

13) Create a new column in Grapher (or Excel) entitled ln A. Have Grapher (or Excel) calculate the natural log, ln of each activity value. Right-click in the graph and in the Graph Options requester, select Axes Options. Click in the box beside Right Y-Axis above the right column, label the axis ln A, and select ln A under Data Set.
Plot ln A vs. t. You should have a nice descending straight line.

Use the linear fit option to draw the line of best fit for this data. Record the slope provided as the "λ from graph."

14) Compare your two values for λ by determining their percent difference.

Table 1 Radioactive Half-Life

Operating Voltage _____ V

Background Radiation _____ counts/min

t Time at start of interval (min)	N Counts in 15 sec	
0.0		
1.0		
2.0		
3.0		
4.0		
5.0		
6.0		
7.0		
8.0		
9.0		
10.0		
11.0		
12.0		

Counter

START

STOP → Record

CLEAR

Half-life for Ba-137m

Time from initial activity, Ao to $\frac{1}{2}Ao$ _____ min   Time from $\frac{1}{2}Ao$ to $\frac{1}{4}Ao$ _____ min

Average experimental half-life, $t_{1/2}$ _____ min   Accepted half-life: 2.6 _____ min

Percent error in half-life _____ %

   Show calculations for percent error in $t_{1/2}$ here.

Decay constant, λ for Ba-137m

Decay constant, λ from average $t_{1/2}$ _____ min^{-1} λ from graph _____ min^{-1}

Percent difference in decay constant _____ %

   Show calculations for percent difference in λ here.

Q1. Define the following terms.

Threshold voltage _____

_____

Half-life, $t_{1/2}$ _____

_____

Decay constant, λ _____

_____

Q2. In the discussion of the theory of the minigenerator a problem was presented along with a follow-up question.

"This brings up the question of where isotopes with short half-lives come from. Shouldn't any that ever existed be pretty-well gone? The isotope that you'll be using both answers this question and solves the problem in the previous paragraph."

How does the Cs-137 → Barium-137 system solve this problem?

_____

_____

_____

Q3. Using the decays scheme used in this lab, explain what an isomer is. _____

_____

_____

Q4. The activity of a radioactive sample is the number of decays per minute that occur in the sample. We measured this by detecting the radiation entering the Geiger tube window. Explain why our readings were actually only a small fraction of the actual activity. Also, why did that not seem to matter in the determination of Barium-137m's half-life? Refer to either your A vs. t or your ln A vs. t graph in your explanation.

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Q5. The Strontium-90 isotope that we used in this lab has a half-life of 28.1 years. In our calculations we used a .10 Curie sample of Sr-90. That is, we used enough Sr-90 to have an activity of .10 Ci. One Ci is an activity of 3.7×10^{-10} disintegrations/s. Calculate the activity of this sample when used by students 281 years from now.